Simultaneous Signaling*

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Abstract

In both university and job application and search processes, an applicant to a particular institution may be able to increase the probability of receiving an offer by demonstrating his or her interest in that institution. We characterize the optimal set of institutions to which a candidate should demonstrate interest (i.e., send costly signals) from among the set of institutions to which a market participant has applied. In a simultaneous-decision model, we demonstrate that a greedy algorithm – one which is simple to characterize and easy to understand – implements the optimal decision rule. Our work generalizes Chade and Smith (2006). Following our characterization of the optimal mechanism, we perform a comparative statics exercise that lends insight into the determination of the optimal set of institutions to which a candidate chooses to signal.

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1 Introduction

The college application and search process and the job application and search process each involves not only a candidate's choice of the set of schools or institutions to apply, but also the set of schools or institutions to signal interest. For example, in the job market for economics Ph.D.s organized by the American Economic Association, a job market candidate not only chooses the set of schools to which he or she applies, but also the set of (two) schools to which he or she sends, through the Association, signals of interest. Similarly, in the college admissions process, applicants have the option to send costly signals to schools, albeit not by a formal institutional process. Examples of such signals are attending campus tours and information sessions, making phone calls and sending e-mail inquiries, all of which require time and effort from the applicants. Some schools track this information to evaluate the candidates' interest in the school. In this paper, we characterize the optimal set of institutions to which a candidate should demonstrate interest (i.e., send costly signals) from among the set of institutions to which a market participant has applied.

Applicants send costly signals because universities and employers want these candidates who demonstrate their interest. In the market for Ph.D. economists, Coles et al. (2010a), with data from the American Economic Association, examine the effectiveness of the AEA signaling mechanism; and in the college admissions process, Dearden, Li, and Meyerhoefer (2011), in their empirical section, with data from a highly-selective medium-sized university, examine the effectiveness of "demonstrated interest."

The Chronicle of Higher Education, 2010, asks the following questions: "How many applicants would turn down a super-selective, big-name college to attend a somewhat less-selective, less-famous one? How do you know whether a student considers your college a top choice or a 'safety school'? How does an applicant's sense of 'fit' with a college relate not only to matriculation, but also retention?" The article continues, "In recent years, such questions have prompted American admissions teams to look more closely at 'demonstrated interest,' the popular term for the contact students make with a college during the application process, such as by visiting the campus, participating in an interview, or e-mailing an admissions representative." (Hoover, 2010)

In theoretical analyses of market signaling, Avery and Levin (2010), Coles et al. (2010b), and Kushnir (2010) examine signaling and institutional offer game-theoretic models. While these models offer insights into signaling decisions, they are restrictive in that each applicant is permitted to send only one signal. Given the established empirical importance of signaling, an interesting issue is the characterization of optimal signaling decisions by a decision maker in a model in which the choice of the number of signals and which schools to signal is endogenous.

In this paper, we consider a setting in which a decision maker — either a student applying to colleges

and universities or a worker applying for jobs — has submitted applications to various institutions. After submitting the applications, the decision maker has the option to send a costly signal to each institution. We analyze an individual's simultaneous choice about the set of institutions to signal. In doing so, we generalize the results of Chade and Smith (2006). Chade and Smith analyze the problem of characterizing the set of institutions to which a decision maker optimally applies, assuming that not applying to an institution results in zero probability of an offer from that institution. Their model, however, cannot be used to describe the signaling choice problem because in the context of signaling, assuming a zero probability of receiving an offer following no signal is unrealistic. To consider the signaling decision problem, we generalize their model by permitting the probability of receiving an offer to be positive (or possibly zero) following no signal.

We demonstrate that the optimal algorithm to solve the decision maker's problem identified in Chade and Smith (2006) for their specific environment also is optimal in this more general environment. Incidentally, they generalize Stigler (1961). This optimal decision rule is a greedy algorithm, which they appropriately term for an economics audience a marginal improvement algorithm (MIA). This algorithm has the advantage of being simple; that is, it is easy to characterize and to understand.

The key to Chade and Smith's proof of the optimal of the marginal improvement algorithm is the "downward recursive" nature of the decision maker's gross payoff function. (In their model, as in ours, the decision maker's utility equals gross payoff less cost.) In our more-general environment, however, the decision maker's gross payoff function is no longer downward recursive. But, as we demonstrate, a monotonic transformation of the gross payoff function does satisfy this property. We use the downward recursive nature of this monotonic transformation in the proof of the optimality of the marginal improvement algorithm.

After we establish that the MIA as optimal in our signaling setting, we examine two issues about the optimal set. The first is about the aggressiveness of the optimal set of schools that the decisions maker chooses to signal. Specifically, we address the question about whether the necessary simultaneity of the signaling decisions in the college admissions process, for example, encourages the decision maker to signal highly-selective schools. We demonstrate that the result that best portfolio of signals is more aggressive that the set of best options taken individually extends to our signaling environment. Furthermore, we demonstrate that the following result extends to our signaling environment: The optimal portfolio of schools to signal is upwardly diverse.

Our second question, in the context of college admissions, is about the effect of changes in a school's selectivity on the decision maker's choice of whether to signal that school. We demonstrate that as a school becomes more selective in its admissions decisions (without an increase in a student's utility of the school), a student tends to shy away from signaling the school. The contrapositive is true as well. We would like to note that to address this question, we change admissions probabilities in a manner that cannot be done in

the Chade and Smith's simultaneous search model.

With few exceptions, our sequence of lemmas and theorems in establishing the optimality and properties of the marginal improvement algorithm is identical to the sequence of Chade and Smith. We would like to note, however, that our proofs are new (Theorem 1 for example) or we altered the Chade and Smith proofs for our more-general environment. As we noted above, the comparative statics of the optimal set is original to our analysis.

2   The Model

2.1   The Signaling Problem

A student (in general, decision maker) has applied to a finite set $N = \{1, 2, ..., N\}$ of schools (in general, alternatives).[1] We abuse notation by letting N represent both a natural number and a set. The cardinality of the set of subsets of N is 2^N. The student's problem in this model is to choose a subset of schools to which he or she signals. Let $f : 2^N \longmapsto \Re_+$ be a non-decreasing function. Interpret $f(S)$ as the expected gross payoff to the student of signaling a subset S of the schools. We let ρ_A^S denote the probability of being rejected by each of the schools in $A, A \subseteq N$, given that the student signals each of the schools in $S, S \subseteq N$. We let $\rho_A \equiv \rho_A^\emptyset$ denote the probability of being rejected by each of the schools in A, given the student has signaled none of the schools in A. Note: we use the absence of a superscript in ρ_A to indicate that the student has signaled none of the schools in the set A. We assume for each school $i \in N, \rho_i \geq \rho_i^i$. Furthermore, $\rho_A^S = (\prod_{i \in A \wedge S} \rho_i^i)(\prod_{i \in A-S} \rho_i)$. We label the schools by the student's ex post utility, $u_1 > u_2 > ... > u_N$. The student's gross ex ante expected payoff of signaling the schools in S schools is:

$$f(S) = \sum_{k=1}^{N} \rho_{[1,k)}^S (1 - \rho_k^S) u_k. \tag{1}$$

We assume that the cost of signaling a portfolio of schools, S, is a function of the cardinality of S, $c(|S|)$, where $|S|$ denotes the cardinality of S. We assume that c is increasing and convex and that $c(0) = 0$.

We examine the choice of S to maximize the student's net payoff, $v(S) = f(S) - c(|S|)$. For the problem to be interesting, we assume that $f(i) - c(1) > f(\emptyset)$ for at least one i. As in Chade and Smith, our analysis requires the consideration of a subset $D \subseteq N$. For $D \subseteq N$, let $\Sigma^*(D)$ solve

$$\max_{S \subseteq D} v(S), \tag{2}$$

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[1] As in Chade and Smith (2006), we avoid using set notation by writing for example $i = \{i\}$, $A + B = A \cup B$, $A - B = A \setminus B$, $A \wedge B = A \cap B$, and $(i, j) = \{k \in N \mid i < k < j\}$.

and denote $\Sigma^* \equiv \Sigma^*(N)$.

In our analysis, we also make use of a special case in which the student signals exactly n schools from the set of alternatives D. For this special case, the cost function is $c(|S|) = 0$ for $|S| \le n$ and $c(|S|) = \infty$ for $|S| > n$. For this case, we let $\Sigma_n(D)$ denote the solution to (2). Further, we define $\Sigma_n \equiv \Sigma_n(N)$.

From f, we define the function, $g : s^N \longmapsto \Re_+$, which equals the student's net expected payoff of signaling of signaling S less his or her or expected payoff of signaling the null set, $\emptyset$. That is, $g(S) \equiv f(S) - f(\emptyset)$. In our investigation of the solution to (2), which involves the maximization of $f(S) - c(|S|)$, we analyze the function g.

2.2   Properties of the Payoff Functions, f and g

We define that U *is above* L, written $U \sqsupseteq L$, if the worst prize in U is better than the best prize in L. The function g is *downward recursive* (DR). Specifically, given the set of signaling options U and L in N that satisfy $U \sqsupseteq L$, we have that g is DR:

$$g(U + L) = g(U) + \frac{\rho_U^U}{\rho_U} g(L). \tag{3}$$

In the appendix, we prove that g is DR.

We also have that g satisfies the following multiplicative property. For any $U \sqsupseteq M \sqsupseteq L$, we have:

$$\begin{aligned}
g(U + M + L) &= g(U + M) + \frac{\rho_{U+M}^{U+M}}{\rho_{U+M}} g(L). \\
&= \left(g(U) + \frac{\rho_U^U}{\rho_U} g(M) \right) + \frac{\rho_{U+M}^{U+M}}{\rho_{U+M}} g(L).
\end{aligned}$$

Because $\dfrac{\rho_S^S}{\rho_S} \le 1$ for any S and is multiplicative, this function is decreasing in S.

Even though the function g is a monotonic transformation of f, and g is DR, the function f is not DR. We demonstrate by means of a counterexample that the function f is not DR. In this counterexample, consider $N = 2$. To demonstrate that f is not DR, set $f(1+2) = f(1) + \alpha f(2)$ and solve for α. In doing so, we have

$$\alpha = \frac{\rho_1^1(\rho_2 - \rho_2^2)u_2}{(1 - \rho_1)u_1 + \rho_1(1 - \rho_2^2)u_2},$$

which indicates that f is not DR.

3 The Solution

In establishing the solution to (2), we begin with properties of the optimal set, then characterize the optimal algorithm.

3.1 Properties of the Optimal Set

Chade and Smith establish a key property of DR functions – *downward maximization*. We establish in Lemma 1 that this property extends to f, which is not DR in our environment. Note that we do use the monotonic relationship between g, which is DR, and f, which is not, in our proof of Lemma 1.

Lemma 1. *Let* $\Sigma_n = U + L$, *where* $U \sqsupseteq L$ *and* L *has* k *elements. Then* $\Sigma_k(D) = L$, *where* D *are those options in* N *that are not better ranked than the best in* L.

The proofs of all lemmas and theorems are in the appendix.

We move on to Lemma 2, where we consider any two alternatives i and j for which the ex post preferences, $u_i > u_j$ (i.e., $i < j$), match the ex ante preferences, $f(i) > f(j)$. For these pairs of alternatives, Lemma 2 states that the marginal values of adding these alternatives to a set $S \subset N - \{i, j\}$ has the same ranking, $MB_i(S) = f(S + i) - f(S) > f(S + j) - f(S) = MB_j(S)$.

Lemma 2. *Assume* $f(i) > f(j)$ *and* $i < j$. *Then the marginal benefits of* i *and* j *are ordered* $MB_i(S) = f(S + i) - f(S) > f(S + j) - f(S) = MB_j(S)$ *for any set* $S \subseteq N - \{i, j\}$.

We demonstrate by means of an example that we cannot apply the Chade and Smith proof of their Lemma 2 to establish our Lemma 2. In this example, consider $N = 3$ in which $f(1) > f(3)$. We put the proof technique of Chade and Smith in the context of this example. For $\rho_1 = \rho_2 = \rho_3 = 1$, Chade and Smith demonstrate in their Lemma 2 that $f(1 + 2) > f(2 + 3)$. To establish this inequality, at one point in their proof, they write the expected utility of the signaling schools 1 and 2 and then using the following suboptimal decision rule for attendance. Choose school 2 if accepted at 2; choose school 1 if accepted at 1 and rejected by 2; and finally choose school 3 if accepted by 3 and rejected by 1 and 2. In a step of their proof, they demonstrate that for the case in which $\rho_1 = \rho_2 = \rho_3 = 1$, this suboptimal policy yields an expected utility that is greater than $f(2 + 3)$.

In the context of this example, to use the Chade and Smith proof technique, the expected utility of the signaling 1 and 2 and using the suboptimal attendance policy less $f(2 + 3)$ must satisfy:

$$(1 - \rho_2^{[1,2]})(f(2) + \rho_2^{[1,2]}((1 - \rho_1^{[1,2]})u_1 + \rho_1^s f(3))) > (1 - \rho_2^{[2,3]})(f(2) + \rho_2^{[2,3]}((1 - \rho_3^{[2,3]})u_3 + \rho_3^{[2,3]} f(3))). \quad (4)$$

We demonstrate by means of this example that (4) does not hold for the case in which $(\rho_1, \rho_2, \rho_3) \neq (1, 1, 1)$.

Continuing with the example, let $u_1 = 2.4$, $u_2 = 2$, and $u_3 = 1.9$. Also let $\rho_1 = 0.9$, $\rho_2 = 0.8$, $\rho_3 = 0.7$, $\rho_1^s = 0.0.8$, $\rho_2^s = 0.7$, $\rho_3^s = 0.6$. We also have on the left-hand side of (4):

$$(1 - \rho_2^{[1,2]})u_2 + \rho_2^{[1,2]}((1 - \rho_1^{[1,2]})u_1 + \rho_1^{[1,2]}(1 - \rho_3)u_3) = 1.2552.$$

We have on the right-hand side of (4):

$$(1 - \rho_1)u_1 + \rho_1((1 - \rho_2^{[2,3]})u_2 + \rho_2^{[2,3]}(1 - \rho_3^{[2,3]})u_3) = 1.2588.$$

Hence, for this example (4) does not hold.

Note that in this numerical example, we have that the sufficient condition of Lemma 2 holds. Specifically, $f(1) = 1.1648 > f(3) = 1.1472$.

Based on this example, we therefore develop in the appendix a new proof of Lemma 2, one that differs from Chade and Smith's proof of their Lemma 2.

Our next lemma yields a simple insight about Σ^*.

Lemma 3. *Assume $f(i) > f(j)$ and $i < j$. If $j \in \Sigma_n(N)$, then $i \in \Sigma_n(N)$.*

3.2 The Optimal Algorithm

We move on to establishing that the following greedy algorithm, by an inductive procedure, identifies Σ^*.

Marginal Improvement Algorithm (MIA):

0. Let $\Upsilon_0 = \emptyset$. Begin with $n = 1$.

1. Choose any $i_n \in \arg\max_{i \in N - \Upsilon_{n-1}} f(\Upsilon_{n-1} + i)$.

2. If $f(\Upsilon_{n-1} + i_n) - f(\Upsilon_{n-1}) < c(n) + c(n-1)$, then stop.

3. Set $\Upsilon_n = \Upsilon_{n-1} + i_n$ and go to step 1.

The MIA works as follows. The student begins by calculating $f(1)$ through $f(n)$, and includes the option, i, with the greatest value in S. The student then recalculates, determining $f(i + j)$ for each $j \in N - i$. He or she adds the option j with the greatest value, $f(i + j)$. The student continues until he or she hits the point where for each $k \notin S$, $f(S + k) - f(S) < c(|S + 1|) - c(|S|)$.

For the function f, the marginal benefit of adding j to choice set S is decreasing in S. Specifically, for $S \subset S'$, we have:

$$f(S + j) - f(S)$$

$$= \rho^S_{[1,j)} \left(\sum_{t=j}^{N} \rho^S_{(j,t]}(\rho_j - \rho^j_j)(u_t - u_{t+1}) \right)$$

$$\geq \rho^{S'}_{[1,j)} \left(\sum_{t=j}^{N} \rho^{S'}_{(j,t]}(\rho_j - \rho^j_j)(u_t - u_{t+1}) \right)$$

$$= f(S' + j) - f(S')$$

We therefore have:

Lemma 4. *The function* $f : 2^N \mapsto \Re$ *is submodular, and thus has diminishing returns.*

Our primary result is:

Theorem 1. *The MIA implements the optimal set* Σ^* *for problem* (2) *with* $D = N$.

4 Two Properties of the Optimal Set

4.1 Portfolio Choices are More Aggressive than Top Singletons

One important property of the optimal set, established by Chade and Smith, is that the portfolio choices are more aggressive than the top singletons. To compare the aggressiveness of the optimal set to the top singletons, Chade and Smith employ a vector first-order stochastic dominance (FSD) criterion. The set $S \subseteq N$ is *more aggressive* than the same-size set $S' \subseteq N$ in the sense of FSD when $s_{(i)} \leqslant s'_{(i)}$ for all i, where $s_{(i)}$ is the ith best school in S and $s'_{(i)}$ in S'. Following Chade and Smith, we write this as $S \succeq S'$ and as $S \succ S'$ if also $S \neq S'$. As an example, $\{1, 2\} \succ \{2, 3\}$.

We define the set of top singletons as the set $Z_{|\Sigma^*|} \subseteq N$ as the options with the $|\Sigma^*|$ highest payoffs $f(i)$.

Theorem 2. *The best portfolio* Σ^* *is more aggressive than the best singletons* $Z_{|\Sigma^*|}$.

While $f(i)$ — the net benefit of signaling only i — captures cross-college external effects of applications, it misses the greater cross-college external effects as captured in (1) of signaling the schools in $|\Sigma^*| - i$. Capturing these greater cross-college external effects through simultaneous search encourages students to signal schools that are higher ranked, to which students tend to assign greater utility but also tend to have smaller probability benefits from signaling, $(\rho_i - \rho^i_i)$. This result explains why the very best U.S.

universities either are or could be inundated with demonstrated interest. As a result of the cost of tracking and processing these signals, some universities have decided to eliminate demonstrated interest as a factor in their admissions decisions. For example, "Duke does not take demonstrated interest into account when evaluating applications. Although we are glad that you may have visited our campus or asked us questions about the school, demonstrated interest is not an advantage in the admissions process."[2]

4.2   Portfolio Choices are Upwardly Diverse

Chade and Smith also demonstrate that in some scenarios students in their application choices tend to gamble upward by choosing to apply to a discretely higher college than the others in the student's portfolio. That is, it is not true in general that students apply to an interval of schools $[i, j]$. We demonstrate that this same result holds for the optimal signaling portfolio.

To illustrate this point, consider a scenario with $N - 1$ copies of college j and one copy of college i. Comparing colleges i and j, we consider $j > i$ and $f(j) > f(i)$. The scenario has constant marginal cost, $\bar{c}$.

In this scenario, the student's payoff of signaling school i and $n - 1$ copies of school j less the student's payoff of signaling n copies of school j is

$$(\rho_i - \rho_i^i) \left[u_i - \left(1 + (\rho_j^j)^{n-1}(\rho_j)^{N-n-1}(\rho_j - \rho_j^j) \right) u_j \right].$$

As n becomes very large, $\rho_j^{j^{n-1}}$ vanishes. Hence, as n becomes very large, this expression is positive. For $\bar{c}$ sufficiently small, but not too small, it is optimal to signal n schools. If so, the student prefers signaling school i and $n - 1$ copies of school j to signaling n copies of j. Hence, there exists $n < N$ for which the student signals school i before exhausting all possibilities of signaling school j. Chade and Smith note that by continuity this result holds even for the case which the copies of school j are not identical. That is, the result holds if there is a sufficiently dense collection of similar colleges and a diverse group of higher-ranked schools.

5   Comparative Statics

We analyze the effect of a change in a school's selectivity on the student's signaling decision. In addressing this issue, which we do in Theorem 3, we change only the probabilities of admission, conditional on signaling and not signaling the school; we do not change the student's utility of the school. Furthermore, to change school i's selectivity, we change the admissions probabilities of school i, ρ_i^S and ρ_i^{S+i} $(i \notin S)$, so that $\rho_i^S - \rho_i^{S+i}$

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[2]See http://www.admissions.duke.edu/faq/index.html, "Applying for Admission", question 5. Accessed April 8, 2012.

does not change.

With this specific change in selectivity, the student's problem changes in an interesting way that adds intricacies to the comparative static analysis. Specifically, as we demonstrate in the proof to Theorem 3, changing ρ_i^S and ρ_i^{S+i}, without changing $\rho_i^S - \rho_i^{S+i}$, leaves the student's marginal benefit of signaling school i, $MB_i(S) = f(S+i) - f(S)$, unchanged. However, the marginal benefit of signaling each other school, $MB_j(S) = f(S+j) - f(S)$, for each S, does change. Therefore, while the changes in ρ_i^S and ρ_i^{S+i} may affect Σ^*, Theorem 3 answers the question about whether these changes in probabilities affects the decision to signal school i.

To examine the effect of the change in a school's selectivity, we introduce additional notation. Let f^ρ denote the expected utility and $\Sigma^{\rho*}$ denote the optimal set under under the probability structure, $\rho = (\rho_1, ..., \rho_N, \rho_1^1, ..., \rho_N^N)$. In Theorem 3, we compare the optimal choice sets under two different probability structures: $\rho = (\rho_1, ..., \rho_N, \rho_1^1, ..., \rho_N^N)$ and $\varrho = (\rho_1, ..., \varrho_i, ..., \rho_N, \rho_1^1, ..., \varrho_i^i, ..., \rho_N^N)$ for which $\varrho_i > \rho_i$, $\varrho_i^i > \rho_i^i$, and $\varrho_i - \varrho_i^i = \rho_i - \rho_i^i$.

Theorem 3. *Consider $\rho = (\rho_1, ..., \rho_N, \rho_1^1, ..., \rho_N^N)$ and $\varrho = (\rho_1, ..., \varrho_i, ..., \rho_N, \rho_1^1, ..., \varrho_i^i, ..., \rho_N^N)$ for which $\varrho_i > \rho_i$, $\varrho_i^i > \rho_i^i$, and $\varrho_i - \varrho_i^i = \rho_i - \rho_i^i$.*

$$(i) \text{ If } i \notin \Sigma^{\rho*}, \text{then } i \notin \Sigma^{\varrho*}.$$

$$(ii) \text{ If } i \in \Sigma^{\varrho*}, \text{then } i \in \Sigma^{\rho*}.$$

Suppose a student does not signal school i. According to Theorem 3 (i), if i becomes more selective, then the student continues not to signal the school. However, if i becomes less selective, the student may switch to signaling the school. We construct a simple example to demonstrate the possible switch. Let $N = 2$, $u_1 = 4.3$, $u_2 = 4.0$, $\rho_1^1 = \rho_1 - 0.2$, $\rho_2^2 = 0.3$, and $\rho_2 = 0.5$. In this case, $f(1) - f(0) = 0.46$ and $f(2) - f(0) = 0.8\rho_1$. Suppose the cost of signaling one school is zero while the cost of signaling two schools is prohibitive. Hence, if $\rho_1 > 0.575$, then the student signals school 2 and not school 1. If school 1 becomes sufficiently less selective, so that $\rho_1 < 0.575$ (and therefore $\rho_1^1 < 0.375$), then the student switches to signaling school 1.

Alternatively, suppose the student signals school i. According to Theorem 3 (i), if i becomes less selective, then the student continues to signal the school. However, if i becomes more selective, the student may switch to not signaling the school. Continuing with the previous example, if school 1 becomes sufficiently selective so that $\rho_1 > 0.575$, then the student signals school 1. If school 1 becomes sufficiently less selective, so that $\rho_1 < 0.575$, then the student switches to signaling school 2.

Next, we demonstrate in Corollary 1 that the Theorem 3 result also holds for the case in which the increase in the probability associated with signaling is smaller for the more-selective incarnation of the school.

Corollary 1. *Consider* $\rho = (\rho_1, ..., \rho_N, \rho_1^1, ..., \rho_N^N)$ *and* $\varrho = (\rho_1, ..., \varrho_i, ..., \rho_N, \rho_1^1, ..., \varrho_i^i, ..., \rho_N^N)$ *for which either* $\varrho_i > \rho_i$, $\varrho_i^i > \rho_i^i$, *and* $\varrho_i - \varrho_i^i < \rho_i - \rho_i^i$

$$(i) \text{ If } i \notin \Sigma^{\rho*}, \text{then } i \notin \Sigma^{\varrho*}.$$

$$(ii) \text{ If } i \in \Sigma^{\varrho*}, \text{then } i \in \Sigma^{\rho*}.$$

6   Conclusion

We demonstrated that a greedy algorithm implements the optimal set for a simultaneous signaling problem, a problem that applies most directly to college admissions and job search processes. In characterizing the optimal algorithm, we extended the analysis of Chade and Smith (2006) to an environment in which non-inclusion in a choice set results possibly in a positive probability of acceptance — either university admission or a job offer.

In our model, the aggressiveness and upward diversity of the optimal signaling set suggest that college and job applicants tend to include highly-selective schools and jobs in their choices of institutions to signal. However, our comparative statics results indicate that the applicant's utility of these highly-selective institutions, and not the high selectivity itself, attracts the signals.

Our identification of the optimal algorithm in a simultaneous signaling problem is part of two larger research issues. The first is the determination of the optimal algorithm for a simultaneous application and signaling problem. As an example of this problem, a high school senior must decide not only the set of schools to which he or she applies, but also the subset (of those to which he or she sends applications) of schools to which the applicant sends a signal. Our preliminary results on this problem indicate that a greedy algorithm is not optimal. The second problem is the determination of the acceptance probabilities (with and without signals) in a college or job matching problem. Dearden et al. (2011) examine, both theoretically and empirically, whether signaling has a heterogeneous effect in a population of applicants. In particular, this model examines the determination of equilibrium acceptance probabilities as a function of applicant SAT scores. Their analysis demonstrates that the positive effect of a signal on acceptance probabilities is increasing in SAT scores. The results of their theoretical model are somewhat restrictive, however, because the model has only two selective schools and the number of signals sent by a particular applicant is exogenous. The examination of a general model with N schools and a general signaling cost function could yield interesting results.

APPENDIX

Proof of DR Property. We begin with:

$$g(U + L) = f(U + L) - f(\emptyset) = \sum_{t=1}^{N} \rho_{[1,t)}^{U+L}(1 - \rho_t^{U+L})u_t - f(\emptyset). \tag{A-1}$$

We partition $\{1, ..., N\}$ into two sets $\{1, ..., l_1 - 1\}$ and $\{l_1, ..., N\}$, where l_1 is the best alternative in L, and rewrite (A-1) to obtain:

$$= \sum_{t=1}^{l_1-1} \rho_{[1,t)}^{U}(1 - \rho_t^{U})u_t + \sum_{t=l_1}^{N} \rho_{[1,t)}^{U+L}(1 - \rho_t^{U+L})u_t - f(\emptyset). \tag{A-2}$$

We add and subtract the same expression from (A-2) to obtain:

$$= \sum_{t=1}^{l_1-1} \rho_{[1,t)}^{U}(1 - \rho_t^{U})u_t + \sum_{t=l_1}^{N} \rho_{[1,t)}^{U+L}(1 - \rho_t^{U+L})u_t - f(\emptyset).$$

$$+ \sum_{t=l_1}^{N} \rho_{[1,t)}^{U}(1 - \rho_t)u_t - \sum_{t=l_1}^{N} \rho_{[1,t)}^{U}(1 - \rho_t)u_t. \tag{A-3}$$

We rearrange the terms in (A-3) and use these equalities: first, $\rho_t^{U+L} = \rho_t^{L}$ for any $t \in [l_1, N]$; second, $\rho_t = \rho_t^{U}$ for any $t \in [l_1, N]$; third, $\rho_U^{U}\rho_{[1,t)}^{L} = \rho_U \rho_{[1,t)}^{U+L}$; and fourth $\rho_U^{U}\rho_{[1,t)} = \rho_U \rho_{[1,t)}^{U}$. In doing so, we obtain:

$$= \sum_{t=1}^{N} \rho_{[1,t)}^{U}(1 - \rho_t^{U})u_t - f(\emptyset)$$

$$+ \frac{\rho_U^{U}}{\rho_U}\left(\sum_{t=l_1}^{N} \rho_{[1,t)}^{L}(1 - \rho_t^{L})u_t - \sum_{t=l_1}^{N} \rho_{[1,t)}(1 - \rho_t)u_t \right). \tag{A-4}$$

We use the fact that the first $l_1 - 1$ terms of $f(L)$ and $f(\emptyset)$ are the same and thus cancel each other in $g(L) = f(L) - f(\emptyset)$ to find:

$$= g(U) + \frac{\rho_U^{U}}{\rho_U}g(L). \tag{A-5}$$

which shows that Property (3) holds. □

Proof of Lemma 1. By contradiction. Assume $\Sigma_k(D) \neq L$. That is, there exists an $S \subseteq D$ such that $|S| = k$ and $f(S) > f(L)$. By definition of the set U, $U \sqsupseteq D$, which implies $U \sqsupseteq S$. The DR property states that

$g(U + S) = g(U) + \frac{\rho_U^U}{\rho_U} g(S)$. Then,

$$f(S) > f(L) \Leftrightarrow g(S) > g(L) \Leftrightarrow \frac{\rho_U^U}{\rho_U} g(S) > \frac{\rho_U^U}{\rho_U} g(L).$$

Therefore,

$$\begin{aligned} g(U + S) &= g(U) + \frac{\rho_U^U}{\rho_U} g(S) \\ &> g(U) + \frac{\rho_U^U}{\rho_U} g(L) \\ &= g(U + L). \end{aligned}$$

However, $\Sigma_n = U + L$ implies $f(U + L) \geq f(U + S)$ and thus $g(U + L) \geq g(U + S)$, which is a contradiction. Therefore $\Sigma_k(D) = L$. $\qquad\qquad\square$

Proof of Lemma 2. We start with $f(i) - f(j)$ and gradually build $f(S+i) - f(S+j)$. In our notation, we let $u_{N+1} = 0$, which is a dummy option that we include to preserve the integrity of our expression. We express $f(i)$, the expected utility from signaling school i, as:

$$\begin{aligned} f(i) &= \sum_{t=1}^{i-1} \rho_{[1,t)}\,(1 - \rho_t)u_t + \rho_{[1,i)}\,(1 - \rho_i^i)u_i + \sum_{t=i+1}^{N} \rho_{[1,t)-i}\,\rho_i^i(1 - \rho_t)u_t \\ &= (1 - \rho_1)u_1 + ... + \rho_{[1,i)}(1 - \rho_i^i)u_i + ... + \rho_{[1,N-1]}^i(1 - \rho_N)u_N \\ &= u_1 - \rho_1(u_1 - u_2) - ... - \rho_{[1,i)}\,\rho_i^i(u_i - u_{i+1}) - ... - \rho_{[1,N]-i}\,\rho_i^i(u_N - u_{N+1}). \end{aligned}$$

We now construct $f(i) - f(j)$. In expressing this difference, we can derive from the above expression that the first $min\{i, j\} - 1$ terms of $f(i)$ and $f(j)$ are identical. As $i < j$, the first $i-1$ terms of $f(i) - f(j)$ cancel. Note that in this proof, we partition the signaling set S into U, M, and L, where $U \subseteq [1, i)$, $M \subseteq (i, j)$, and $L = \subseteq (j, N]$. We have that $f(i) - f(j)$ is expressed as follows:

$$f(i) - f(j)$$
$$= \rho_{[1,i)}\left(\sum_{t=i}^{j-1} \rho_{(i,t]}(\rho_i - \rho_i^i)(u_t - u_{t+1}) + \sum_{t=j}^{N} \rho_{(i,t]-j}\,(\rho_i\rho_j^j - \rho_i^i\rho_j)(u_t - u_{t+1})\right) > 0. \qquad \text{(A-6)}$$

Continuing with the examination of $f(i) - f(j)$, we multiply the inequality by $\frac{\rho_U^U}{\rho_U} \in [0, 1]$ and obtain the

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following:

$$= \rho_{[1,i)}^U \left(\sum_{t=i}^{j-1} \rho_{(i,t]}(\rho_i - \rho_i^i)(u_t - u_{t+1}) + \sum_{t=j}^{N} \rho_{(i,t]-j}\left(\rho_i \rho_j^j - \rho_i^i \rho_j\right)(u_t - u_{t+1}) \right) > 0. \tag{A-7}$$

Note that $\rho_i - \rho_i^i \geq 0$ and $u_t - u_{t+1} > 0$. Therefore, all terms of the summation on the left (that is, for $t = i, ..., j-1$) are positive. Also note that $\rho_i \rho_j^j - \rho_i^i \rho_j$ can be positive or negative. Therefore, all terms of the summation on the right (that is, $t = j, ..., N$) will either be positive or negative. Our assumption that $f(i) - f(j) > 0$ implies that the cumulative sum up to $n \in i, ..., N$ is always positive, because the negative terms in (A-7) occur only starting with term $t = j$.

We use the following property in the next step of the proof.

Property 1. *Let $A, B \in \mathbb{R}$ such that $A > 0$ and $A + B > 0$, and let $r \in [0, 1]$. Then we have*

$$A + rB > 0, \tag{A-8}$$

because if $B > 0$ then $A + rB > r(A + B) > 0$, and if $B \leq 0$ then $A + rB > A + B > 0$.

Next, let A be the cumulative sum up to $t = n - 1$, B be the sum over the remaining terms, and r be $\frac{\rho_n^s}{\rho_n} \in [0, 1]$ for some $n \in i, ..., N$. For $A > 0$, $A + B > 0$ and $r \in [0, 1]$, we can apply (A-8) in Property 1 to the following. We let $M = S \wedge (i, j)$. We multiply each term in (A-7) for $t > n$ and $t \in S$ by $r = \frac{\rho_n^s}{\rho_n}$ iteratively until all n are accounted for:

$$\rho_{[1,i)}^U \left(\sum_{t=i}^{j-1} \rho_{(i,t]}^M(\rho_i - \rho_i^i)(u_t - u_{t+1}) + \sum_{t=j}^{N} \rho_{(i,t]}^{M+L}\left(\rho_i \rho_j^j - \rho_i^i \rho_j\right)(u_t - u_{t+1}) \right) > 0.$$

We rearrange the left-hand-side by splitting the factors $(\rho_i - \rho_i^i)$ and $(\rho_i \rho_j^j - \rho_i^i \rho_j)$, and also by taking the outer factors, $\rho_{[1,i)}^U$, inside. In doing so, we obtain:

$$= \sum_{t=i}^{N} \rho_{[1,t]}^S \rho_j^j(u_t - u_{t+1}) - \sum_{t=i}^{N} \rho_{[1,t]}^S \rho_i^i(u_t - u_{t+1}) > 0. \tag{A-9}$$

Finally, add and subtract $u_1 - \sum_{t=1}^{t=i-1} \rho_{[1,t]}^U(u_t - u_{t+1})$. (Note that this term is 0 if $U = \emptyset$.) In doing so, we see that the left-hand-side of (A-9) is equal to $f(S + i) - f(S + j)$. Therefore, $f(S + i) - f(S + j) > 0$. $\square$

Proof of Lemma 3. By contradiction. Assume $j \in \Sigma_n(N)$, but $i \notin \Sigma_n(N)$. Let $S \equiv \Sigma_n(N) - \{j\}$. Then by Lemma 2, $f(S + i) > f(S + j)$. This implies $f(S + i) > f(\Sigma_n(N))$ which leads to a contradiction as $\Sigma_n(N)$ is the optimal solution. $\square$

Proof of Theorem 1. Assume we are given $\Sigma_n(N)$. Let j be the lowest-ranked (i.e., largest-indexed) option in $\Sigma_{n+1}(N) - \Sigma_n(N)$. Note that such a j must exist because $\Sigma_{n+1}(N)$ has one more option than does $\Sigma_n(N)$. That is, $|\Sigma_{n+1}(N)| = |\Sigma_n(N)| + 1$. Let S be the subset of options in $\Sigma_{n+1}(N)$ that are ranked lower than j. Note that by definition of j, S is common to both optimal sets. That is, $S \subseteq \Sigma_{n+1}(N) \cap \Sigma_n(N)$. Let $\{i_1, \ldots, i_{n-|S|}\}$ be the options in $\Sigma_n(N)$ that are better ranked than j.

We will iteratively show that j being in the optimal set $\Sigma_{n+1}(N)$ implies that each $i_t, t \in \{1, \ldots, n-|S|\}$ is also in the optimal set $\Sigma_{n+1}(N)$.

Let us start with $t = 1$. We set the rejection probabilities $\rho_k = \rho_k^k$ for all $k \in S$. By Lemma 1, we have $\Sigma_1([1, i_1]) = i_1$. This implies $f(i_1) > f(j)$. By Lemma 3, because $i_1 < j$ and $j \in \Sigma_{n+1-|S|}([1, i_1])$, we have $i_1 \in \Sigma_{n+1-|S|}([1, i_1])$.

For the remaining t, we set the rejection probabilities $\rho_{i_{t-1}} = \rho_{i_{t-1}}^{i_{t-1}}$. By Lemma 1, we obtain $\Sigma_1([1, i_t]) = i_t$. This implies $f(i_t) > f(j)$. By Lemma 3, since $i_t < j$ and $j \in \Sigma_{n+1-|S|-(t-1)}([1, i_t])$, we have $i_t \in \Sigma_{n+1-|S|-(t-1)}([1, i_t])$.

We increment t. When $t = n - |S|$, we obtain $i_{n-|S|} \in \Sigma_2([1, i_{n-|S|}])$. As j is also in the optimal set, this implies $\Sigma_2([1, i_{n-|S|}]) = \{j, i_{n-|S|}\}$, which in turn, implies $\Sigma_{n+1}(N) = \Sigma_n(N) + \{j\}$.

We also have $\Sigma_1(N) = \arg\max_{k \in N} f(k)$.

Because $\Sigma_1(N) = \arg\max_{k \in N} f(k)$ and $\Sigma_{n+1}(N) = \Sigma_n(N) + \{j\}$, the MIA returns $\Upsilon_n(N) = \Sigma_n(N)$ for each n.

To complete the proof, the stopping rule in the MIA is optimal because $c(n)$ in convex in n and f by Lemma 4 has diminishing returns — $f(A + k) - f(A)$ is decreasing in A for any $k \notin A \subseteq N$. Furthermore, because c is a function of only the cardinality of N, $\Sigma^* = \Sigma_n(N)$ □

Proof of Theorem 2. The singletons are chosen according to $(f(1), ..., f(N))$. By Lemma 2, for any portfolio S excluding i, j, we have that $MB_i(S) > MB_j(S)$. Therefore, we have the following. Consider k that is in σ^* but is not among the top $|\sigma^*|$ singletons. By Lemma 3, we cannot have j in σ^* while i is not. Furthermore, suppose j is among he top $|\sigma^*|$ singletons but is not in σ^*. By Lemma 2, we must have that $k < j$. □

Proof of Theorem 3 and Corollary 1. Statement (ii) is the contrapositive of statement (i). Therefore, (ii) has the same truth value as (i). We only prove (i). In our proof, we begin by establishing four properties about the relationship between f^ρ and f^ϱ. With these properties in place, we prove the claim in (i), namely if $i \notin \Sigma^{\rho*}$, then $i \notin \Sigma^{\varrho*}$.

For some $i \in N$, let $\varrho_i - \rho_i = \delta$ and $\varrho_i^i - \rho_i^i = \delta$, and note that $\delta > 0 + \gamma$, for which $\delta > 0$ and $\gamma \geq 0$. Note: for the proof of Theorem 3, $\gamma = 0$; and for the proof of Corollary 1, $\gamma > 0$. We now establish the first of four relationships of f^ρ and f^ϱ.

Claim 1: If $f^\rho(S+j) > f^\rho(S+i)$ for some $j < i$ and $S \subseteq N - \{i,j\}$, then $f^\varrho(S+j) > f^\varrho(S+i)$.

We prove this claim using (A-6):

$$f^\varrho(S+j) - f^\varrho(S+i)$$

$$= \rho_{[1,j)}^S \left(\sum_{t=j}^{i-1} \rho_{(j,t]}^S (\rho_j - \rho_j^j)(u_t - u_{t+1}) + \sum_{t=i}^{N} \rho_{(j,t]-i}^S (\rho_j \varrho_i - \rho_j^j \varrho_i)(u_t - u_{t+1}) \right)$$

$$= \rho_{[1,j)}^S \left(\sum_{t=j}^{i-1} \rho_{(j,t]}^S (\rho_j - \rho_j^j)(u_t - u_{t+1}) + \sum_{t=i}^{N} \rho_{(j,t]-i}^S (\rho_j(\rho_i^i + \delta + \gamma) - \rho_j^j(\rho_i + \delta))(u_t - u_{t+1}) \right)$$

$$\geq f^\rho(S+j) - f^\rho(S+i) + \rho_{[1,j)}^S \left(\sum_{t=i}^{N} \rho_{(j,t]-i}^S \delta(\rho_j - \rho_j^j)(u_t - u_{t+1}) \right)$$

$$> f^\rho(S+j) - f^\rho(S+i) > 0. \tag{A-10}$$

This completes the proof of Claim 1.

Next, we move on to our second relationship between f^ρ and f^ϱ. For this relationship, we consider three schools i, j and k, for which $j > i$ and also $k \neq j$. (Recall $\varrho_i > \rho_i$, $\varrho_i^S > \rho_i^S$, and $\varrho_i - \varrho_i^i \leq \rho_i - \rho_i^i$.)

Claim 2: If $f^\rho(S+j) > f^\rho(S+k)$ for some $j > i$ and $S \subseteq N - \{j,k\}$, then $f^\varrho(S+j) > f^\varrho(S+k)$.

To prove Claim 2, we first consider $k < j$. Note that the calculations below depend on whether i is greater than, less than, or equal to k. Referring again to (A-6), we have:

$$f^\rho(S+j) - f^\rho(S+k)$$

$$= \rho_{[1,k)}^S \left(\sum_{t=k}^{j-1} \rho_{(k,t]}^S (\rho_k^k - \rho_k)(u_t - u_{t+1}) + \sum_{t=j}^{N} \rho_{(k,t]-j}^S (\rho_k^k \rho_j - \rho_k \rho_j^j)(u_t - u_{t+1}) \right).$$

We use this expression, (A-6), to build $f^\varrho(S+j) - f^\varrho(S+j)$ from $f^\rho(S+j) - f^\rho(S+j)$.

If $k > i$, when building $f^\varrho(S+j) - f^\varrho(S+j)$ from $f^\rho(S+j) - f^\rho(S+j)$, the only variable that changes in (A-6) is $\rho_{[1,k)}^S$. Therefore, $sign|f^\varrho(S+j) - f^\varrho(S+j)| = sign|f^\rho(S+j) - f^\rho(S+j)|$.

If $k = i$, only the last term in the parentheses of (A-6) changes. We obtain:

$$f^\varrho(S+j) - f^\varrho(S+k) = f^\rho(S+j) - f^\rho(S+k)$$

$$+ \rho_{[1,k)}^S \left(\sum_{t=j}^{N} \rho_{(k,t]-j}^S [\delta(\rho_j - \rho_j^j) + \gamma \rho_j](u_t - u_{t+1}) \right) > 0.$$

Finally, if $k < i$, both terms in the parentheses of (A-6) change. Note that the terms in parentheses

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below are greater than 0. We obtain:

$$f^{\varrho}(S+j) - f^{\varrho}(S+k) = f^{\rho}(S+j) - f^{\rho}(S+k)$$

$$+ \rho^{S}_{[1,i)-k}\delta \left(\sum_{t=i}^{j-1} \rho^{S}_{(i,t]}(\rho^{k}_{k} - \rho_{k})(u_t - u_{t+1}) + \sum_{t=j}^{N} \rho^{S}_{(i,t]-j} (\rho^{k}_{k}\rho_j - \rho_k\rho^{j}_{j})(u_t - u_{t+1}) \right)$$

$$+ \rho^{S}_{[1,i)-k}\gamma \left(\sum_{t=i}^{j-1} \rho^{S}_{(i,t]}(\rho^{k}_{k} - \rho_k)(u_t - u_{t+1}) \right) > 0.$$

For the case in which $k > j$, a change in ρ_i or ρ^s_i does not affect the sign of $f(S+j) - f(S+k)$. Therefore, claim 2 holds for $k > j$. This completes the proof of Claim 2.

Claim 3: Consider a set $S, S \subset \Sigma^{\rho*}$. If $f(S+j) > f(S+k)$ for all $k \in N - S$, then $j \in \Sigma^{\rho*}$.

To prove Claim 3, observe that we can solve for the optimal choice set by the following two-step process. First, create a new probability structure $\hat{\rho}$ by setting ρ^{k}_{k} equal to ρ_k for each $k \in S$, where S is a set in $\Sigma^{\rho*}$. Second, for this new probability structure $\hat{\rho}$, solve for $\Sigma^{\hat{\rho}*}(N - |S|)$. Because we proved that MIA finds the optimal choice given any $0 \leq \rho^{k}_{k} \leq \rho_k \leq 1$, the MIA can be used to find $\Sigma^{\hat{\rho}*}(N - |S|)$. Then, finding j such that $f(S+j) > f(S+k)$ for all k is the next step of MIA applied to the probability structure $\hat{\rho}$, which completes the proof of Claim 3.

Claim 4: The marginal benefit of adding i to choice set S is no greater for the probability profile ϱ than it is for the probability profile ρ.

To prove Claim 4, we have:

$$f^{\varrho}(S+i) - f^{\varrho}(S)$$

$$= \rho^{S}_{[1,i)} \left(\sum_{t=i}^{N} \rho^{S}_{(i,t]}(\rho_i + \not\delta - \rho^{i}_i - \not\delta - \gamma)(u_t - u_{t+1}) \right)$$

$$\leq f^{\rho}(S+i) - f^{\rho}(S)$$

We are finished with the four claims.

We now proceed to prove (i) by contradiction. Assume $i \notin \Sigma^{\rho*}$ and $i \in \Sigma^{\varrho*}$. Consider the MIA. We start with $\emptyset$, and step-by-step build our optimal set $\Sigma^{\varrho*}$ under the probability structure, ϱ. In the optimal set, $\Sigma^{\rho*}$, let $j_m \in \arg\max_{j \in N} f(\Sigma^{\rho}_{m-1} + j)$ denote the school added at stage j of the MIA under the probability structure ρ.

We begin with step 1. If $j_1 < i$, by Claim 1 and Lemma 3, $j_1 \in \Sigma^{\varrho*}$. If $j_1 > i$, then by Claim 2, $j_1 \in \Sigma^{\varrho*}$. We now have that $j_1 \in \Sigma^{\varrho*}$.

We continue building the optimal set $\Sigma^{\varrho*}$ and arrive at step m, $1 < m < n$, where we attempt to add i

to $\Sigma^{\varrho*}$. However, by the same proof as in step 1 and Claim 3, $j_m \in \Sigma^{\varrho*}$, $j_m \neq i$. Therefore, we cannot add i before $n+1^{\text{st}}$ iteration.

If we add i at an iteration $r > n$, then by Claim 4 we obtain:

$$f^{\varrho}(\Sigma^{\rho} + \{j_{n+1}, \ldots, j_{r-1}\} + i) - f^{\varrho}(\Sigma^{\rho} + \{j_{n+1}, \ldots, j_{r-1}\})$$

$$\leq f^{\rho}(\Sigma^{\rho} + \{j_{n+1}, \ldots, j_{r-1}\} + i) - f^{\rho}(\Sigma^{\rho} + \{j_{n+1}, \ldots, j_{r-1}\}). \tag{A-11}$$

By diminishing returns (i.e. Lemma 4), we obtain:

$$f^{\rho}(\Sigma^{\rho} + \{j_{n+1}, \ldots, j_{r-1}\} + i) - f^{\rho}(\Sigma^{\rho} + \{j_{n+1}, \ldots, j_{r-1}\}) \leq f^{\rho}(\Sigma^{\rho} + i) - f^{\rho}(\Sigma^{\rho}). \tag{A-12}$$

Finally, by the optimality of Σ^{ρ} under initial probability structure ρ, we obtain:

$$f^{\rho}(\Sigma^{\rho} + i) - f^{\rho}(\Sigma^{\rho}) \leq c(n+1) - c(n). \tag{A-13}$$

Putting together (A-11), (A-12), and (A-13), we have that i is not added to $\Sigma^{\varrho*}$ by the MIA in any iteration $r > n$.

We now have that i is not included in any iteration of the step-by-step building of $\Sigma^{\varrho*}$ by the MIA. Hence, we have a contradiction. $\qquad\square$

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