

WILEY

American Finance Association

A Note on Quantity Versus Price Risk and the Theory of Financial Intermediation

Author(s): Stephen D. Smith, Deborah Wright Gregory and Kathleen A. Weiss

Source: *The Journal of Finance*, Vol. 42, No. 5 (Dec., 1987), pp. 1377-1383

Published by: [Wiley](#) for the [American Finance Association](#)

Stable URL: <http://www.jstor.org/stable/2328533>

Accessed: 10/06/2014 17:29

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Wiley and American Finance Association are collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Finance*.

<http://www.jstor.org>

A Note on Quantity versus Price Risk and the Theory of Financial Intermediation

STEPHEN D. SMITH, DEBORAH WRIGHT GREGORY, and
KATHLEEN A. WEISS*

MODELING THE BEHAVIOR OF financial intermediaries under conditions of uncertainty has a long and varied history.¹ Moreover, it continues to be a topic of considerable interest because appropriate public policy decisions depend, in part, on the assumed characteristics of firms participating in the market. One central point of debate, as presented by Sealey [3], involves the potentially contrasting behavioral implications associated with assuming that intermediaries are price setters in their deposit markets, as opposed to quantity setters operating under conditions of perfect competition. He addresses, among other things, the important question of whether the supply of financial intermediation is altered by the inclusion of quantity risk. Sealey argues that, in contrast to earlier work by Pyle [1], one cannot determine how the degree of association between asset rates and the quantity of deposits influences the amount of lending by intermediaries.

The purpose of this note is twofold. First, we provide conditions under which the ambiguity between Sealey's and Pyle's works may be resolved. Second, we argue that the results from the two models are symmetric. We use symmetry here to denote the fact that the conditions on association that, for example, discourage intermediation are the same in both models when viewed in the context of revenues and costs. However, the two frameworks require opposite conditions on the association between rates and quantities to achieve identical results concerning the relationship between revenues and costs.

The approach used involves constructing a scenario that makes the Sealey model almost identical to that used earlier by Pyle except for the rate versus quantity setting issue. We then show that a positive association between asset rates and the quantity of deposits actually *discourages* lending but promotes the gathering of more deposits. We then provide a rationale for why this is consistent with Pyle's conjecture that a negative association discourages intermediation in the pure rate-uncertainty case. Interestingly, we also argue that, while a negative association will promote lending by depository intermediaries under certain conditions, the result in this case is generally ambiguous, as is the case when a positive association is analyzed in the Pyle model.

The paper is structured as follows. In Section I, we present the basic model, derive our results, and discuss their implications. Section II contains concluding remarks.

* Smith is from the College of Management, Georgia Institute of Technology; Gregory and Weiss are from the Department of Finance, Insurance and Real Estate, Graduate School of Business, University of Florida. While we would like to thank an unknown reviewer and George Morgan for helpful comments, any errors or ambiguities are our responsibility.

¹ See Santomero [2] for an extensive review and critique of the existing literature.

I. Association and Intermediation

The model and notation used here are almost identical to those in Sealey [3], except for an assumption that the real resource cost function for deposits is linear. This provides for a more direct comparison with the portfolio theoretic models, which remain unaltered by adding such a constant marginal cost function.² We specifically assume that the intermediary chooses a quantity of loans, L , and a deposit rate, R_D , to maximize the expected value of a von Neumann-Morgenstern utility function, $U(\cdot)$,³ defined over profits, π (with $U'(\pi) \geq 0$, $U''(\pi) \leq 0$, and $U'''(\pi) \geq 0, \forall \pi$), where

$$\pi = (R_L - p)L - (R_D - p)D(R_D, \mu) - C_L(L) - C_D(D(R_D, \mu)), \quad (1)$$

R_L is the (random) return on loans, p is the known borrowing/lending rate in the market for purchased funds, and $D(R_D, \mu)$ is the random quantity of deposits purchased, given a deposit rate of R_D and a random element, μ . We assume that

$$\frac{\partial D(R_D, \mu)}{\partial R_D} > 0 \quad \text{and} \quad \frac{\partial D(R_D, \mu)}{\partial \mu} > 0,$$

$C'_L(L) > 0$, $C''_L(L) \geq 0$, and $C'_D(D(R_D, \mu)) = d > 0$, where d is some known constant. The cost-function assumption with respect to loans provides for a determinant loan size (when the inequality $C''_L(L)$ is strict) even in the absence of strict risk aversion, in contrast with Pyle's model.

However, since the deposit rate will be determinant even under risk neutrality, we use the simplifying assumption that $C''_D(\cdot) = 0$ in order to disentangle the pure quantity versus price risk element from random marginal real resource costs. This would not be possible if $C_D(D(R_D, \mu))$ were of a more general form. Therefore, the objective function becomes

$$\begin{aligned} \max E[U(\pi)] \\ &= E[U[(R_L - p)L - (R_D - p)D(R_D, \mu) - C_L(L) - C_D(D(R_D, \mu))]] \\ &= G(L, R_D), \end{aligned} \quad (2)$$

which is identical to equation (8) in Sealey [3]. The first-order conditions are given by

$$\begin{aligned} \frac{\partial E[U(\pi)]}{\partial L} \\ &= E[U'(\pi)(R_L - p - C'_L(L))] = 0 = G_L(L, R_D), \end{aligned} \quad (3a)$$

$$\begin{aligned} \frac{\partial E[U(\pi)]}{\partial R_D} \\ &= E[U'(\pi)((p - R_D - d)\partial D/\partial R_D - D(R_D, \mu))] = 0 = G_{R_D}(L, R_D). \end{aligned} \quad (3b)$$

² While we recognize the importance of both potential economies and diseconomies of scale (as well as scope) in the production of loans and servicing of deposits, we ignore these issues in order to clearly focus on the question of quantity versus price risk.

³ The appropriateness of this assumption versus that of value maximization (see Sealey [4]) remains an open issue. Santomero [2] provides a thorough discussion of the controversy.

Equations (3a) and (3b) can be rewritten as

$$G_L = E(U'(\pi))E(R_L - p - C'_L(L)) + \text{cov}(U'(\pi), R_L) = 0 \quad (4a)$$

$$\begin{aligned} G_{R_D} = E(U'(\pi))[(p - R_D - d)E(\partial D/\partial R_D) - E(D(R_D, \mu))] \\ + (p - R_D - d)\text{cov}(U'(\pi), \partial D/\partial R_D) \\ - \text{cov}(U'(\pi), D(R_D, \mu)) = 0, \end{aligned} \quad (4b)$$

where $\text{cov}(\cdot)$ is the covariance operator. Furthermore, it can be verified from (3a) that $(p - R_D - d)$ must be positive if R_D is to be an equilibrium deposit rate (since $\partial D/\partial R_D$, $D(R_D, \mu)$, and $U'(\pi)$ are positive by assumption).

We now provide two results associated with the covariances that form the basis of our analysis concerning association and intermediation in a deposit rate-setting environment.

PROPOSITION: *If marginal utility is bonded below and $U'''(\pi) \geq 0$, the covariance in (4a) is negative for any positive value of L if loan rates and deposit flows are non-negatively associated.⁴*

Proof: The covariance in (4a) may be rewritten as

$$E_{R_L}\{\text{cov}[E[U'(\pi) | R_L], R_L]\},$$

where

$$E[U'(\pi) | R_L] = \int_0^\infty U'(\pi) dF(\mu | R_L), \quad (5)$$

where $F(\mu | R_L)$ is the conditional distribution of μ given R_L . We wish to show that⁵

$$\frac{dE[U'(\pi) | R_L]}{dR_L} \leq 0.$$

Integrating by parts over μ yields (given $U'(\infty) = 0$)

$$E[U'(\pi) | R_L] = - \int_0^\infty U''(\pi) F(\mu | R_L) (p - R_D - d) \frac{\partial D}{\partial \mu} d\mu. \quad (6)$$

⁴ The concept of association used here is analogous to that used by Pyle. In particular, if $F(X | Y)$ is the conditional distribution of X given Y ,

$$\frac{dF(X | Y)}{dY} \cong 0 \rightarrow \frac{dE[X | Y]}{dY} \cong 0 \rightarrow \text{cov}(X, Y) \cong 0,$$

but not the other way around, where $E(\cdot | \cdot)$ is the conditional expectation of X given Y . Therefore, by positive association we mean that the conditional distribution function of X , given Y , is declining in Y , for all Y .

⁵ For brevity we suppress the notation of $R_L = R_L^*$ or $\mu = \mu^*$ as realization of the random variables R_L and μ , respectively.

Differentiating (6) with respect to R_L provides

$$\frac{dE[U'(\pi) | R_L]}{dR_L} = (R_D + d - p) \left[\int_0^\infty [U'''(\pi)LF(\mu | R_L) + U''(\pi)F'_{R_L}(\mu | R_L)] d\mu \right]. \quad (7)$$

Now $R_D + d - p < 0$. Furthermore, if μ and R_L are non-negatively associated, $F'_{R_L}(\mu | R_L) \leq 0$. Therefore, (7) is strictly decreasing for $L > 0$. Finally, it is known that the covariance of a decreasing and an increasing function of a random variable is negative. Q.E.D.

COROLLARY: Given that the proposition holds, the covariance in (4b) is positive if

$$\frac{\partial^2 D}{\partial R_D \partial \mu} \leq 0.$$

Proof: The method of proof is analogous to that used for the proposition. In particular, the covariance in (4b) can be written as

$$\text{cov}(U'(\pi), X(\mu)) = E_\mu[\text{cov}(E(U'(\pi) | \mu), X(\mu))], \quad (8)$$

where

$$X(\mu) = (p - R_D - d)\partial D/\partial R_D - D(R_D, \mu).$$

Now,

$$\frac{dX(\mu)}{d\mu} = (p - R_D - d) \frac{\partial^2 D}{\partial R_D \partial \mu} - \frac{\partial D}{\partial \mu} < 0 \quad (9)$$

if $\partial^2 D/\partial R_D \partial \mu < 0$. Moreover, for $L > 0$, after integrating by parts over R_L , we have

$$E[U'(\pi) | \mu] = - \left[\int_0^\infty U''(\pi)F(R_L | \mu) dR_L \right] L. \quad (10)$$

Differentiating (10) with respect to μ yields

$$\frac{dE[U'(\pi) | \mu]}{d\mu} = L \left[\int_0^\infty [U'''(\pi)(p - R_D - d)\partial D/\partial \mu F(R_L | \mu) + U''(\pi)F'_\mu(R_L | \mu)] dR_L \right], \quad (11)$$

which is negative if $F'_\mu(R_L | \mu) \leq 0$. Finally, the covariance of two decreasing functions of a random variable is known to be positive. Q.E.D.

The importance of the proposition and corollary involves the fact that they imply, *ceteris paribus*, that the slopes of the objective function, (4a) and (4b), will be smaller and larger, respectively, if loan rates and deposit flows are positively associated. This implies that, under the conditions of the proposition and corol-

lary, intermediaries will offer fewer loans and higher deposit rates vis-à-vis the case where interest rate and liquidity risk are independent.

The intuition behind the proposition follows from the fact that, when the firm faces both rate and liquidity risk, a positive association implies that revenues and costs will be negatively associated. The firm will need to borrow heavily in the more expensive slack market in situations where lending profits are low. This, coupled with the basic risk of lending, discourages lending for risk-averse decision makers. However, this positive association between rate and liquidity risk will not necessarily shrink the overall size of intermediaries. In fact, the corollary shows that, as long as the deposit rate is lowest when the random element of deposits is highest,⁶ the intermediary will actually offer a higher deposit rate in order to bolster deposits. These additional funds are not used to make loans, but rather to invest in the purchased-funds market, which, by assumption, is riskless. Therefore, intermediation on the liability side of the balance sheet will actually increase when loan rates and deposit quantities are positively associated. This positive impact makes sense because the random state of nature is supplying a relatively large quantity of deposits to lend in the purchased-funds market in precisely those instances when changes in the deposit rate are having little impact in raising additional deposits and vice versa. Of course, the converse would be true (the deposit rate would be lower) in the opposite case.⁷ Such a situation would result in a shrinkage of the depository intermediary on both sides of the balance sheet.

It can be shown that the above results are consistent with the case of a negative association between asset and liability rates in the pure rate-uncertainty case considered by Pyle.⁸ This follows from the fact that revenues and costs will also be negatively associated in this situation. The basic intuition discussed above applies in this case as well. Therefore, the case of rate risk provides an identical answer to that found in the quantity-risk case when the sign of the association is reversed.

Up to this point, we have limited our discussion to the case where deposits and interest rates are positively associated. As noted by Sealey, this corresponds to a business-cycle view of deposits and interest rates. While the question is ultimately an empirical one, the opposite scenario (negative association) is certainly of interest.⁹ A negative association could occur when disintermediation, arising from

⁶ This would be the case with a separable supply function or a situation where changes in R_D bring about smaller changes in the deposit base, the higher the random element of deposits.

⁷ An example of this scenario could arise in the case where the supply function is multiplicative, i.e., $D(R_D, \mu) = \alpha R_D^\beta \mu$. In this case,

$$\frac{dX(\mu)}{d\mu} = (p - R_D - d)\beta\alpha R_D^{\beta-1} - \alpha R_D^\beta,$$

where $X(\mu) = (p - R_D - d)(\partial D/\partial R_D) - D(R_D, \mu)$. Now $dX(\mu)/d\mu > 0$ if and only if $\beta(p - R_D - d) > R_D$. Note that β is the elasticity of the deposit supply function with respect to R_D , and R_D is one plus the deposit rate. Therefore, if β is sufficiently greater than one, the inequality could possibly go the other way (i.e., the covariance in (4b) could be negative).

⁸ Proof is available upon request.

⁹ We wish to thank the reviewer for bringing up this point.

unexpected changes in interest rates, characterizes an economy. Given an optimal deposit rate, deposits flow out of the depository intermediary as market rates unexpectedly rise and vice versa when rates fall. Our analysis suggests that such a situation may actually promote more lending by depository firms, *ceteris paribus*. The intuition here is one of diversification. When rates and deposit flows are negatively associated, the intermediary needs to borrow less in the more expensive purchased-funds market in those situations where revenues from lending are low. In this case, loan revenues and deposit costs are positively associated, which in turn reduces the risk of the firm. Moreover, while this intuition can be formalized for small lending levels, a general proof of this case is not possible.¹⁰ This follows from the fact that the basic risk associated with making loans (versus lending in the risk-free market) may outweigh the positive diversification benefits for large enough lending levels. However, it can be shown that the same ambiguity, as well as its resolution for small values of L , occurs in Pyle's analysis when asset and liability rates are positively related. Therefore, the ambiguity does not arise because of the quantity-versus-price-risk issue *per se*.

II. Conclusions

The purpose of this note has been to investigate the role of quantity (or liquidity) versus price risk and the theory of financial intermediation. We specifically show that an intermediary facing quantity uncertainty that is positively associated with interest rates will make fewer loans when compared with a situation where the risks are independent. However, we also argue that the depository firm may not shrink in this case since, under certain conditions, it will offer a higher deposit rate in order to attract additional deposits, which can then be sold in a slack market (e.g., the federal-funds market). Under these conditions, the existence of positively associated interest and liquidity risk leads to a situation where depository intermediaries limit their traditional lending operations while at the same time raising an even larger quantity of funds to sell in the lower risk securities markets (as opposed to traditional loan markets). We also show that a negative association between rates and flows can promote lending by depository intermediaries, other things being the same. The intuition is analogous to the diversification benefits of two negatively associated assets in portfolio theory. Finally, it is noted that the results provided here are consistent with the analysis under pure price risk when the signs of the associations are reversed.

The above results should be of interest to policy makers debating monetary questions such as quantity versus price stabilization and pro versus counter-cyclical intervention. Managers may also find them of use in assessing the risk/return aspects of their asset and liability choices.

¹⁰ Both parts of this remark follow directly from equation (7). At $L = 0$,

$$\frac{dE[U'(\pi) | R_L]}{dR_L} \cong 0$$

if $F_{R_L}(\mu | R_L) \cong 0$. Therefore, the covariance will be positive if $F_{R_L}(\mu | R_L) > 0$ (a negative association). Furthermore, by continuity, the covariance must be positive for some small positive values of L . However, if L is large enough, (7) may be negative if $F_{R_L}(\mu | R_L)$ is positive enough.

REFERENCES

1. D. Pyle. "On the Theory of Financial Intermediation." *Journal of Finance* 26 (June 1971), 737-47.
2. A. Santomero. "Modeling the Banking Firm: A Survey." *Journal of Money, Credit, and Banking* 16 (November 1984), 576-602.
3. C. Sealey. "Deposit-Rate Setting, Risk Aversion, and the Theory of Depository Financial Intermediaries." *Journal of Finance* 35 (December 1980), 1139-54.
4. ———. "Valuation, Capital Structure, and Shareholder Unanimity for Depository Financial Intermediaries." *Journal of Finance* 38 (June 1983), 857-71.