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Efficiency and exclusion in collective action allocations

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Abstract

Suppose that a group of people meet to determine the cost allocation of a collective action, and that each individual's valuation of the collective action is private and independent of all others' values. With this information structure there is an incentive problem—each individual has an incentive to understate value in order to lower his or her cost share of the collective action. This paper examines exclusion from the benefits of a collective action as a tool to reduce the incentive to understate value in collective action negotiations. We (i) identify conditions for which an auction-like mechanism approaches classical ex post efficiency for the allocation of an excludable public good, (ii) characterize ex ante optimal allocation mechanisms, and (iii) identify conditions for which Ramsey pricing is ex ante optimal. © 1997 Elsevier Science B.V.

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1. Introduction

A group of people meet to negotiate the cost allocation of a collective action and also to decide who in the group receives the benefits.[1] For example, a professional group

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[1] According to Sandler (1992), p. 1, ''Collective action arises when the efforts of two or more individuals are needed to accomplish an outcome. Activities that involve the furtherance of the interests or well-being of a group are often examples of collective action.'' In this paper, we consider one particular type of collective action. An individual's benefit from participating in the collective action is independent of the number and characteristics of the other individuals who participate. Hence, we do not consider direct externalities. However, the cost-per-participant in the collective action is decreasing in the number of participants. Thus, we do consider collective actions, such as public goods, that arise from the gains of sharing costs.

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wants to engage a distinguished and costly speaker.[2] How does the group finance the speaker (given that the revenues from the event must cover the cost)? Which people in the group attend the event? In this paper, we address these questions by considering a model of collective action allocation in which each individual's valuation of the collective action is private information and independent of all others' valuations. With this valuation and information structure, each person has an incentive to understate his or her own valuation in order to reduce his or her monetary contribution to fund the action, and thereby receive the benefits of the action and free-ride from others' contributions.

Rob (1989) and Mailath and Postlewaite (1990a) (hereafter denoted by R-M-P) examine the collective action problem. They demonstrate that under very mild conditions the free-rider problem is so severe that the probability of the group taking the collective action goes to zero as the group becomes very large. The R-M-P conditions are that each individual's value of a collective action is private information and independent, each individual must voluntarily contribute to the cost of the collective action, there is a balanced budget for the project, and no individual can be excluded from the provision of the collective action.[3] This result places such dismal prospects on the possibility of taking collective actions that the natural question is how can the R-M-P requirements be relaxed in order to promote more explicit collective action allocations.

This paper investigates one possible tool to increase the efficiency of collective action allocations: the possible exclusion of some individuals from the benefits of the collective action. Our objective is to examine both the efficiency properties of allocation mechanisms and the actual price and exclusion features of the efficient mechanisms.

If the probability of participating in a collective action is increasing in an individual's reported value, then the individual has an increased incentive not to understate value. The efficiency benefit of potential exclusion is then derived from this increased incentive not to understate value. Exclusion also carries an efficiency cost. If someone's value of the collective action is greater than the marginal cost of provision and the person is excluded, then some of the potential economic surplus is lost. Hence, there is a trade-off between the efficiency costs and benefits of exclusion in the design of efficient allocation mechanisms. However, in terms of the mechanism design problem, possible exclusion adds an additional degree of freedom to the design of efficient allocation mechanisms. Therefore, say the mechanism designer's objective in the selection of an efficient mechanism is to maximize the expected economic surplus. Then for any set of weights, the introduction of possible exclusion as a design instrument only increases the maximum expected surplus.

Economists have investigated the advantage of exclusion for solving the free-rider problem in collective action negotiations. Silva and Kahn (1993) examine the optimal amount of exclusion for quasi-public goods for the case in which valuations are common knowledge. They show that even though individual valuations are common knowledge, people can shirk on payments for the collective action (thereby creating moral hazard).

[2] It is important to note that we consider a static problem in which each person has a unit inelastic demand for the collective action.

[3] Roberts (1976), obtains a similar result for the allocation of pure public goods under stronger feasibility requirements that those of R-M-P.

Exclusion from the provision of the collective action is a punishment for shirking on the tax payments. Silva and Kahn determine the optimal amount of costly expenditures on exclusion devices. Moreover, closely related to our investigation of exclusion and collective actions is the literature on local public goods and clubs (introduced by Tiebout, 1956, and Buchanan, 1965, respectively) which examines the types and number of people to include in the provision of various collective actions. Recent papers (see Silva–Kahn for references) on local public goods and club goods assume that individual preferences are common knowledge and instead examine congestion effects and pricing mechanisms. However, our primary focus in this paper is on exclusion and pricing (or taxation) as a preference revelation mechanism. For related work on preference revelation see Moulin (1994), who examines one specific form of cost sharing (serial cost sharing) and Mailath and Postlewaite (1990b), who consider a network of workers negotiating the split of the profits of a new firm formed by the workers and formally examine the conditions under which possible exclusion creates efficient decisions for very large groups. Also, Deb and Razzolini (1994, 1995) examine strategy-proof mechanisms for the allocation of excludable public goods.

Exclusion is an increasingly popular policy instrument for determining the use of excludable public goods. Examples include highway tolls, user fees for public parks, public sewer fees, and cable television fees. The application that partially inspired our analysis is a U.S. (Congressional Budget Office, 1992) report on user fees for funding highways, airways, and waterways. The main policy suggestion of the CBO report is that marginal-cost pricing, compared to current pricing practices such as average-cost pricing, promotes greater economic efficiency of infrastructure use. One important difficulty with marginal-cost pricing is recognized by the CBO report—the potential funding shortfalls (when fixed cost is especially large compared to marginal cost). In fact, Theorem 1 of this paper demonstrates that the marginal-cost pricing of collective actions will generally result in a budget deficit. In particular, this theorem proves that if the cost-per-person included is decreasing with respect to the size of the user group, then the marginal cost of inclusion is less than this average cost, and marginal-cost pricing cannot fund the project. If marginal-cost pricing is infeasible, what allocation mechanism is then optimal among the class of feasible mechanisms?

2. Overview of the paper

The feasibility of an economic mechanism to allocate the costs and benefits of a collective action depends on the particular legal requirements of the mechanism, the financial requirements to fund the collective action, and on the information possessed by the potential consumers. In some legal contexts, individuals have the right to choose whether to consume the collective action and pay for its use. In others, a majority of a community can vote for the construction of a collective action and impose cost shares on all individuals in a community.

The imposition of a less-than-unanimity rule may solve the free-rider problem. For example, by relaxing the participation constraint and imposing a simple majority rule, allocation mechanisms that were not feasible under unanimity rule may now be approved by a majority of the group. In order to provide the collective action as the group size gets very large, Mailath and Postlewaite (1990a, Corollary to Theorem 2) demonstrate that an extreme rent must be collected from the minority. In particular, the minority in total receives utility equal to the cost of the collective action less the sum of all individuals' minimum possible valuations. Another implication of Mailath and Postlewaite (1990a), Corollary to Theorem 2, is that the elimination of the balanced budget requirement also yields classical ex post efficient allocation mechanisms. However, the Budget imbalance is extreme—it equals the sum of the individuals' minimum possible valuations less the cost of the project. Green and Laffont (1979) demonstrate that in the absence of an individual rationality constraint, in the limit, classical ex post efficiency can be attained in a dominant strategy, ex post balanced budget mechanism.[4]

In this paper, we consider the case in which individuals have the right to voluntarily choose whether to consume the collective action and pay a corresponding fee. Also, in the institutional context we examine, an allocation mechanism must have a balanced budget. As for the information possessed by the potential consumers, we consider the case in which each individual has private information about his or her valuation, and valuations are independent. In direct allocation mechanisms, we require that individuals honestly report their valuations to a coordinator. In our model, a direct allocation mechanism is then feasible if individuals voluntarily participate in the mechanism, the mechanism has a balanced budget, and individuals honestly report their valuations to a coordinator.

The various forms of feasibility depend on the information that the individuals possess when they are asked whether they choose to participate in the allocation, whether the individuals have access to a bank that is willing to cover any funding shortfalls and retain any funding surpluses, and the equilibrium requirements for the honest reporting of valuations. One particular form of feasibility—which we call weak incentive feasibility—requires that each individual agree to the terms of allocation mechanism after learning only his or her own valuation, the allocation have an expected balanced budget (where the expectation is over all possible valuations), and honest reporting of valuations form a Bayesian–Nash equilibrium. Strong incentive feasibility requires that each individual agree to the terms of allocation mechanism after learning his or her own valuation, the allocation have a balanced budget, and honest reporting of valuations form a dominant strategy equilibrium. Weak incentive feasibility, strong incentive feasibility, and the other forms of feasibility are formally defined in the next section.[5]

We demonstrate in Section 4 that a weakly incentive feasible, classical ex post efficient mechanism (i.e, a mechanism that captures the entire economic surplus) does not exist. We therefore turn to the examination of other mechanisms. In Section 5, we characterize a strongly incentive feasible auction-like mechanism. For excludable public

[4] Green and Laffont (1979) consider the case of a pure public good in which $c(n, m)$ does not necessarily approach zero. Our Theorem 3 considers the case of an excludable public good in which $c(n, m)$ is independent of n.

[5] Note that for the case of pure public good allocations, R-M-P consider weakly incentive feasible mechanisms, while Roberts (1976) considers strongly incentive feasible mechanisms.

good problems and the sharing of a fixed cost, Theorem 2 demonstrates that this mechanism approaches classical ex post efficiency as the number of individuals approaches infinity. While the auction-like mechanism is simple, universal (i.e., it requires no statistical information about valuations), and has very attractive incentive properties, it is inefficient. We therefore turn, in Section 6, to examine information constrained efficient mechanisms.

In Section 6 we characterize the class of allocation mechanisms (i.e., α^* mechanisms) that maximize the expected economic surplus among the class of weakly incentive feasible mechanisms. We also consider excludable public goods—one particular class of collective actions—and identify conditions for which the expected surplus per individual in the economy is increasing in the size of the economy.

In Section 7 we examine a Ramsey-pricing mechanism. We identify conditions for which the Ramsey-pricing mechanism maximizes the sum of the expected economic surplus of the collective action. As the size of the economy approaches infinity, each person's valuation of the project becomes insignificant compared to the aggregate valuation. Therefore, for very large economies, if any one person announces a low willingness to pay, then this person barely affects the probability the action is taken and lowers his or her contribution. This means that in the limit ($n=\infty$) to prevent people from lying about valuations, an individual's price for using the good is constant in valuation. The mechanism designer then sets ''user'' fees for the individuals. These user fees are determined by the Ramsey formula. Note that one difficulty with Ramsey pricing (and with the α^* mechanism also) for less than a continuum of individuals (i.e., for $n \in N$) is that after demands are realized there may be a budget deficit. For cases in which the consumers cannot run a budget deficit, Ramsey pricing is infeasible. The auction-like mechanism always has an ex post balanced budget and therefore does not have the problem of budget deficits. In Section 8 we discuss efficiency in related environments.

3. The model

An economy of $N=\{1,..., n\}$ individuals consider the construction, monetary payments, and consumption of a collective action. Some individuals may be excluded from the consumption of the collective action. We let $\Omega = \mathcal{P}(N)$ denote the Borel field of N (i.e., the set of all subsets, including the null set, of N). The group of individuals who actually consume the collective action is denoted by ω, $\omega \in \Omega$. The cardinality of Ω is $\#|\Omega| = 2^n$; and the cardinality of ω is denoted by $m_\omega = \#|\omega|$.

Each individual $i \in N$ has a unit-inelastic demand for the collective action, and a valuation v_i for one unit of the collective action.[6] Individual i's valuation, v_i, is private information; and v_i is independently distributed on $[\underline{v}_i, \bar{v}_i]$ according to the strictly positive, continuous density function $f_i(v_i)$, with the corresponding distribution function $F_i(v_i)$. The density function f_i and the distribution function F_i are common knowledge.

[6]Individual i's valuation, v_i, is independent of the number of individuals who consume the collective action. Hence, there is no crowding.

In our model, the cost of providing the good to m individuals can depend not only on that number of individuals but also on the size of the economy, n. For example, with the construction of the main line of a public water system, the cost of providing water service depends on both the entire number of households in a community (i.e., the number of houses that the water system must pass by) and the number of households that receive service. The cost function is represented by c: $N^2 \to \mathbb{R}$ and is expressed as $c(n, m)$. We consider the following class of cost functions.

1. $c(n, m)/m$ is decreasing in m. The cost-per-person included is decreasing with respect to the size of the user group.
2. $c(n, 0) = 0$. The cost of provision to a zero-sized group is zero.
3. $c(n, m+1) - c(n, m) \geq 0$. Marginal cost is non-negative.
4. $\Sigma_{i\in N} \bar{v}_i \geq c(n, n)$ for each $n \in N$ and $\underline{v}_i < c(n, m+1) - c(n, m)$ for each $i \in N$, $n \in \mathbb{N}$ and $m \in \mathbb{N}$. This ensures that the collective action is sufficiently valued and that some valuations are not willing to pay marginal cost.

For a continuum of individuals, $n = \infty$, let γ denote the fraction of individuals who consume the good. The cost per person included is represented by the function ac^{∞}: $[0, 1] \to \mathbb{R}_+$, and is expressed as $ac^{\infty}(\gamma)$.

Before proceeding we would like to note that one particular class of collective actions is excludable public goods. An excludable public good is characterized by the cost function, $c(n, m)$, being constant in m. That is, the cost of provision depends on only the size of the economy, n, and not on the size of the user group, m.

The collective action problem is characterized by three features—the size of the economy, the construction cost, and the probabilistic beliefs of the valuations—$(n, c(n, m), (F_1,...,F_n))$. In some cases, there are Q possible distribution functions on the valuations. The distribution function q, $q \in \{1, 2,...,Q\}$, is denoted by F^q and the corresponding density function by f^q. The fraction of individuals whose valuations are distributed according to the distribution q is denoted by β^q, where $\beta^q \in [0, 1]$, $n\beta^q \in \mathbb{N}$, and $\Sigma_{q=1}^{Q} \beta^q = 1$. For example, consider the funding of a new bridge. All car drivers have valuations distributed according to f^1 and all truck drivers have valuations distributed according to f^2. The fraction of individuals in the population, N, who drive cars is 2/3 and the fraction who drive trucks is 1/3. For this case in which there are Q possible distribution functions, the collective action problem is characterized by $(n, c(n, m), (F^1,...,F^Q), (\beta^1,...,\beta^Q))$.

We consider one particular class of economic mechanisms for the determination of which individuals consume the collective action and of the cost shares. In a direct revelation mechanism each individual $i \in N$, after learning his or her own valuation v_i, reports a valuation $\hat{v}_i$ to a coordinator. This report need not be the individual's true valuation; and the reports by all of the individuals in N are made simultaneously. The coordinator, after receiving all of the reports, states the probability each group of individuals, ω, consumes the collective action and the schedule of payments. By the revelation principle (Myerson, 1991), for any Bayesian Nash equilibrium of any extensive form game there corresponds an equivalent (in payoffs) Bayesian incentive compatible direct revelation mechanism. Moreover, for any dominant strategy equilibrium of any extensive form game, there is a dominant strategy incentive compatible direct revelation mechanism. The power of direct revelation mechanisms is that we can examine only one class of mechanisms, that is direct revelation mechanisms, to determine the possible efficiency of all economic procedures for our collective action problem.

Let ρ_{ω_j} denote the probability that group ω_j and only group ω_j consumes the collective action; and let $\tau_{i_{\omega_j}}$ denote the payment by individual $i \in N$ when group ω_j and only group ω_j consumes the collective action. Formally, a direct revelation mechanism is a pair (ρ, τ), where $\rho: \prod_i [\underline{v}_i, \bar{v}_i] \to \mathbb{R}^{2^n}$ gives the vector of probabilities of inclusion $\rho = (\rho_{\omega_1},...,\rho_{\omega_{2^n}})$; and $\tau: \prod_i [\underline{v}_i, \bar{v}_i] \to \mathbb{R}_+^{n2^n}$ gives the vector of monetary payments $\tau = (\tau_{1_{\omega_1}},...,\tau_{n_{\omega_1}}, \tau_{1_{\omega_2}},...,\tau_{n_{\omega_{2^n}}})$ as a function of the reported valuations. The class of allowable direct revelation mechanisms is general. For example, for a given vector of reported evaluations, the mechanism may specify that 5 individuals will consume the collective action and randomly choose the set of 5 individuals. Moreover, the payments by the individuals may depend on which agents consume the collective action.

For the analysis of direct revelation mechanisms we define the probability that individual i consumes the collective action. To derive that probability we use the characteristic function

$$g_\omega(i) = \begin{cases} 1 & \text{if } i \in \omega, \\ 0 & \text{if } i \notin \omega \end{cases}$$

which states whether individual i is included in ω. The probability that individual i consumes the collective action is then $\Sigma_{\omega \in \Omega}\ \rho_\omega g_\omega(\mathrm{i})$.

Let $v_{-1} \equiv (v_1,...,v_{i-1}, v_{i+1},...,v_n)$ denote the vector of all valuations other than v_i, and let E_{-i} denote the expectation operator over v_{-i}. Let $\tilde{\rho}_\omega(v_i) = E_{-\mathrm{i}}\rho_\omega(v)$ denote the expected probability of provision to group ω, $\Sigma_{\omega \in \Omega}\ g_\omega(i)\tilde{\rho}_\omega(v_i)$ denote the expected probability that individual i consumes the collective action, and $\zeta_i(v_i) = \Sigma_{\omega \in \Omega}\ \tilde{\rho}_\omega(v_i)\tau_{i_\omega}(v)$ denote the expected payment by individual i conditional on v_i. Each individual has a von Neumann–Morgenstern utility function that is additively separable and linear in both money and the value of the collective action. An individual's utility from a mechanism (ρ, τ) is given by $U_i(v_i) = \Sigma_{\omega \in \Omega}\ g_\omega(i)\tilde{\rho}_\omega(v_i)v_i - \zeta_i(v_i)$. We normalize the utility from nonparticipation at zero.

We examine two different types of truthful reporting. The first requires that honest reporting forms a dominant strategy equilibrium to the direct revelation mechanism; that is a mechanism is dominant strategy incentive compatible if for each individual $i \in N$

$$\sum_{\omega \in \Omega} (g_\omega(i)\rho_\omega(v_i, \hat{v}_{-i})v_i - \tau_{i_\omega}(v_i, \hat{v}_{-i})) \geq \sum_{\omega \in \Omega} (g_\omega(i)\rho_\omega(\hat{v}_i, \hat{v}_{-i})v_i$$
$$- \tau_{i_\omega}(\hat{v}_i, \hat{v}_{-i})) \text{ for each } v_i, \hat{v}_i \in [\underline{v}_i, \bar{v}_i] \text{ and for each } \hat{v}_{-i} \in [\underline{v}_1, \bar{v}_1] \times \cdots$$
$$\times [\underline{v}_{i-1}, \bar{v}_{i-1}] \times [\underline{v}_{i+1}, \bar{v}_{i+1}] \times \cdots \times [\underline{v}_n, \bar{v}_n].$$

The second, and weaker, form of incentive compatibility is Bayesian incentive compatibility. Honest reporting must form a Bayesian–Nash equilibrium to the direct revelation mechanism. A mechanism is Bayesian incentive compatible if for each individual $i \in N$

$$U_i(v_i) \geq \sum_{\omega \in \Omega} g_\omega(i)\tilde{\rho}_\omega(\hat{v}_i)v_i - \zeta_i(\hat{v}_i) \text{ for each } \hat{v}_i \in [\underline{v}_i, \bar{v}_i].$$

We require that each individual voluntarily participate in the mechanism (ρ, τ). We examine two different points in time at which the participation decision is made, or equivalently two different individual rationality constraints. The stronger condition requires that, for any realization of the vector of agents' valuations, each individual weakly prefers to participate in the mechanism than not to participate. A mechanism (ρ, τ) is ex post individually rational if for each individual $i \in N$

$$\sum_{\omega \in \Omega} (g_\omega(i)\rho_\omega(v)v_i - \tau_{i_\omega}(v)) \geq 0 \text{ for each } v \in \prod_j [\underline{v}_j, \bar{v}_j].$$

The weaker condition is that an individual, after learning only his or her own valuation, weakly prefers participating in the mechanism than not participating; or the mechanism (ρ, τ) is interim individually rational if for each individual $i \in N$

$$U_i(v_i) \geq 0 \text{ for each } v_i \in [\underline{v}_i, \bar{v}_i].$$

We consider two types of balanced budget conditions. The first, and stronger, requires that after the consumption group, ω, is determined, the payments generated from the mechanism equal the cost. Ex post balanced budget requires

$$\rho_\omega(v) \sum_{i=1}^{n} (\tau_{i_\omega}(v) - c(n, m_\omega)) = 0 \text{ for each } v \in \prod_k [\underline{v}_k, \bar{v}_k] \text{ and for each } \omega \in \Omega.$$

The second, and weaker condition, is relevant if society has access to a risk-neutral credit market. The expected payments equal the expected costs. Ex ante balanced budget requires

$$\int \cdots \int \sum_{\omega \in \Omega} \rho_\omega(v) \sum_{i \in N} (\tau_{i_\omega}(v) - c(n, m_\omega)) \prod_k f_k(v_k)\, \mathrm{d}v_k = 0.$$

We examine two classes of direct mechanisms. A mechanism (ρ, τ) is weakly incentive feasible if it is Bayesian incentive compatible, interim individually rational, and has an ex ante balanced budget. A mechanism (ρ, τ) is strongly incentive feasible if it is dominant strategy incentive compatible, ex post individually rational, and has an ex post balanced budget.

We focus on efficient mechanisms. An allocation mechanism (ρ, τ) is ex ante incentive efficient if no individual's ex ante utility, $\int \mathrm{U}_i(v_i)f_i(v_i)\, \mathrm{d}v_i$, can be increased without either

1. decreasing some other trader's ex ante expected utility, or
2. violating weak incentive feasibility.

We focus on a particular ex ante efficient mechanism: the one that places equal welfare weights on every trader and maximizes the sum of the traders' ex ante expected utilities. This maximization is equivalent to maximizing the sum of all individuals' expected net gains from consumption because the utility function is linearly separable. That is, the ex ante optimal mechanism (ρ, τ) maximizes the expected surplus

$$\int \cdots \int \sum_{\omega \in \Omega} \rho_\omega(v) \sum_{i \in N} (g_\omega(i)v_i - c(n,m_\omega)) \prod_k f_k(v_k)\, \mathrm{d}v_k$$

subject to the constraint that (ρ, τ) is weakly incentive feasible. In contrast a mechanism is classical ex post efficient if it exhausts all of the possible gains to the inclusion of individuals. An allocation mechanism (ρ, τ) is classical ex post efficient if for any $v \in \prod_k [\underline{v}_k, \bar{v}_k]$ no individual's ex post utility, $\Sigma_{\omega \in \Omega} (\rho_\omega(v) g_\omega(i) v_i - \tau_{i_\omega}(v))$, cannot be increased without decreasing some other individual's utility.[7]

4. Weak incentive feasibility and classical ex post inefficiency

To characterize weakly incentive feasible mechanisms, we examine variations on standard mechanism design techniques. Lemma 2 characterizes Bayesian incentive compatible mechanisms and Lemma 3 characterizes weakly incentive feasible mechanisms. The proof of Lemma 2 is standard and omitted.

Lemma 2. (Mailath and Postlewaite, 1990a). *For a collective action problem* $(n, c(n, m), (F_1,\dots,F_n))$, *the mechanism* (ρ, τ) *is Bayesian incentive compatible if and only if* $\Sigma_{\omega \in \Omega}\, g_\omega(i) \tilde{\rho}_\omega(v_i)$ *is non-decreasing in* v_i *and*

$$U_i(v_i) = U_i(\hat{v}_i) + \int_{\hat{v}_i}^{v_i} \sum_{\omega \in \Omega} g_\omega(i) \tilde{\rho}_\omega(v_i)\, \mathrm{d}v_i \text{ for each } v_i, \hat{v}_i \in [\underline{v}_i, \bar{v}_i]. \tag{1}$$

Incentive compatibility implies, from (1), that $U_i(v_i)$ is increasing in v_i, so $U_i(\underline{v}_i) \geq 0$ is necessary and sufficient for interim individual rationality.

Lemma 3. *For a collective action problem* $(n, c(n, m), (F_1,\dots,F_n))$ *suppose* ρ *is a provision rule such that* $\Sigma_{\omega \in \Omega}\, g_\omega(i) \tilde{\rho}_\omega(v_i)$ *is non-decreasing in* v_i *for each i. There exists a contribution scheme* τ *so that* (ρ, τ) *satisfies weak incentive feasibility if and only if*

$$\int \cdots \int \sum_{\omega \in \Omega} \rho_\omega(v) \left(\sum_{i \in N} g_\omega(i) \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) - c(n, m_\omega) \right) \prod_k f_k(v_k)\, \mathrm{d}v_k \geq 0. \tag{2}$$

[7] See Holmström and Myerson (1983), for more on classical and information constrained efficiency requirements.

Proof. Only if.

$$\int \cdots \int \sum_{\omega\in\Omega} \rho_\omega(v)\left(\sum_{i\in N} g_\omega(i)v_i - c(n,m_\omega)\right)\prod_k f_k(v_k)\,\mathrm{d}v_k = \sum_{i\in N}\int U_i(v_i)f_i(v_i)\,\mathrm{d}v_i$$

By Bayesian incentive compatibility

$$=\sum_{i\in N}\left(U_i(\underline{v}_i) + \int_{\underline{v}_i} \sum_{\omega\in\Omega} g_\omega(i)\tilde{\rho}_\omega(v_i)f_i(v_i)\,\mathrm{d}v_i\right).$$

Equating the first and last of these terms, and then requiring an ex ante balanced budget and interim individual rationality, gives us the inequality (2).

If. Suppose that $\rho(\mathrm{v})$ satisfies (2) and $\Sigma_{\omega\in\Omega}\, g_\omega(i)\tilde{\rho}_\omega(v_i)$ is non-decreasing. Consider the payment rule

$$\tau_i(v) = a_i + \sum_{\omega\in\Omega} g_i(\omega)\tilde{\rho}_i(v_i)v_i - \int_{\underline{v}_i}^{v_i} \sum_{\omega\in\Omega} g_i(\omega)\tilde{\rho}_i(\alpha_i)\,\mathrm{d}\alpha_i$$
$$+ \frac{1}{n-1}\sum_{j\neq i}\left[\int_{\underline{v}_i}^{v_j} \sum_{\omega\in\Omega} g_j(\omega)\tilde{\rho}_j(\alpha_j)\,\mathrm{d}\alpha_j - \sum_{\omega\in\Omega} g_j(\omega)\tilde{\rho}_j(v_j)v_j\right].$$

Then

$$U_i(v_i) = -a_i + \int_{\underline{v}_i}^{v_i} \sum_{\omega\in\Omega} g_i(\omega)\tilde{\rho}_i(\alpha_i)\,\mathrm{d}\alpha_i + k_i,$$

where k_i is a constant independent of v_i. Thus, incentive compatibility is satisfied. To check interim individual rationality, set $a_i = k_i$ for $i=1,\ldots,n-1$ and $a_n = -\Sigma_{i\neq n}\, a_i + \Sigma_{\omega\in\Omega}\rho_\omega(v)c(n,\ m)$. Then, $U_i(\underline{v}_i)=0$ for $i=1,\ldots,n-1$, and it is straightforward to check that $U_n(\underline{v}_n)\geq 0$. Also, the ex ante balanced budget requirement is satisfied because $\Sigma_i\ \tau_i(v) = \Sigma_{\omega\in\Omega}\ \rho_\omega(v)\ c(n,\ m)$. Q.E.D.

Remark. It is important to note that possible exclusion relaxes the weak incentive feasibility constraint, (2). Without possible exclusion, we have the restriction that $\rho_\omega(v) = \rho_{\omega'}(v)$ for each $\omega,\ \omega' \in \Omega\backslash\emptyset$. With possible exclusion $\rho_\omega(v)$ does not necessarily equal $\rho_{\omega'}(v)$.

In Theorem 1 we demonstrate that for collective action problems, even if we require only weak incentive feasibility, classical ex post efficiency is unattainable. Theorem 1 generalizes the Myerson and Satterthwaite (1983), and Mailath and Postlewaite (1990a), inefficiency results.

Theorem 1. *For a collective action problem* $(n,\ c(n,\ m),\ (F_1,\ldots,F_n))$, *a classical ex post efficient, weakly incentive feasible allocation mechanism does not exist.*

Proof. We consider one consumption group $\omega = \{1,\ 2,\ldots,m\}$, for $m\in\{1,\ldots,n\}$, and demonstrate that for a classical ex post efficient mechanism, the l.h.s. of expression (2) is negative. In this proof, for notational simplicity $\Delta c \equiv c(n, m) - c(n, m-1)$. Let

$$R = \left\{ (v_1,\dots,v_m): \sum_{i=1}^{m} v_i \geq c, v_i \in [\Delta c, \bar{v_i}] \forall i \in \omega \right\}.$$

The l.h.s. of expression (2) for $\omega \in \Omega$ only is

$$\int \cdots \int_R \left[\sum_{i \in \omega} \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) - c(n, m_\omega) \right] \prod_i f_i(v_i) \, \mathrm{d}v_i \tag{3}$$

Let individual p be "pivotal" in the sense that $\Sigma_{i=p}^m v_i \geq c - (p-1)\Delta c > \Sigma_{i=p+1}^m v_i$. We then define the following two sets of valuations:

$$R_{(p+1)_1} = \left\{ (v_{p+1},\dots,v_m): c - p\Delta c > \sum_{i=p+1}^{m} v_i \geq c - (p-1)\Delta c - \bar{v_p}, v_i \in [\Delta c, \bar{v_i}] \forall i \in \{p+1,\dots,m\} \right\};$$

and

$$R_{(p+1)_2} = \left\{ (v_{p+1},\dots,v_m): c - (p-1)\Delta c > \sum_{i=p+1}^{m} v_i \geq c - p\Delta c, v_i \in [\Delta c, \bar{v_i}] \forall i \in \{p+1,\dots,m\} \right\};$$

Expression (3) is then

$$\begin{aligned}
&\sum_{p=1}^{m-1} \int \cdots \int_{R_{(p+1)_1}} \int_{c-(p-1)\Delta c - \sum_{i=p+1}^m v_i}^{\bar{v_p}} \int_{\Delta C}^{\bar{v}_{p-1}} \cdots \int_{\Delta C}^{\bar{v_1}} \left[\sum_{i=1}^{m} \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) - c \right] \\
&\times \prod_{i=1}^m f_i(v_i) \, \mathrm{d}v_i + \sum_{p=1}^{m-1} \int \cdots \int_{R_{(p+1)_2}} \int_{\Delta c}^{\bar{v_p}} \int_{\Delta c}^{\bar{v}_{p-1}} \cdots \int_{\Delta c}^{\bar{v_1}} \left[\sum_{i=1}^{m} \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) - c \right] \\
&\times \prod_{i=1}^m f_i(v_i) \, \mathrm{d}v_i + \int_{\min(\bar{v}_m, c-(m-2)\Delta c)}^{\bar{v}_m} \int_{\Delta c}^{\bar{v}_{m-1}} \cdots \int_{\Delta c}^{\bar{v_1}} \left[\sum_{i=1}^{m} \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) - c \right] \\
&\times \prod_{i=1}^m f_i(v_i) \, \mathrm{d}v_i.
\end{aligned} \tag{4}$$

To demonstrate that (4) is negative, we use the claim that if the term

$$\int \cdots \int_{R_{(p+1)_1}} \int_{c-(p-1)\Delta c-\sum_{i=p+1}^{m} v_i}^{\bar{v}_p} \int_{\Delta C}^{\bar{v}_{p-1}} \cdots \int_{\Delta C}^{\bar{v}_1} \left[\sum_{i=1}^{m} \left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)} \right) - c \right]$$
$$\times \prod_{i=1}^{m} f_i(v_i)\, \mathrm{d}v_i + \int \cdots \int_{R_{(p+1)_2}} \int_{\Delta c}^{\bar{v}_p} \int_{\Delta c}^{\bar{v}_{p-1}} \cdots \int_{\Delta c}^{\bar{v}_1} \left[\sum_{i=1}^{m} \left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)} \right) - c \right]$$
$$\times \prod_{i=1}^{m} f_i(v_i)\, \mathrm{d}v_i \tag{5}$$

is negative, then

$$\int \cdots \int_{R_{(p+2)_2}} \int_{\Delta c}^{\bar{v}_{p+1}} \int_{\Delta c}^{\bar{v}_p} \cdots \int_{\Delta c}^{\bar{v}_1} \left[\sum_{i=1}^{m} \left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)} \right) - c \right] \prod_{i=1}^{m} f_i(v_i)\, \mathrm{d}v_i \tag{6}$$

is negative. To prove this claim, let A denote the area of integration in (5) and A' denote the area of integration of (6). Then $A' \subset A$. Moreover, if $(v_i,...,v_m) \in A \backslash A'$, then $\Sigma_i v_i > \Sigma_i v_i'$ for any $(v_1',...,v_m') \in A'$. Thus, A contains all of the valuation vectors in A' and also some larger ones. Therefore, if (5) is negative, then (6) is negative.

We integrate (4) by parts and whenever expression (6) appears, we replace it by expression (5). After this process, we derive

$$-\sum_{p=1}^{m-1} p \prod_{i=0}^{p-1} (1 - F_i(\Delta c)) \int \cdots \int_{R_{(p+1)_1}} \left[\sum_{i=p+1}^{m} \left(\frac{1-F_i(v_i)}{f_i(v_i)} \right) + (p-1)\Delta c \right]$$
$$\left[1 - F_p\left(c - \sum_{i=p+1}^{m} v_i \right) \right] \prod_{i=p+1}^{m} f_i(v_i)\, \mathrm{d}v_i - (m-2)[\min\{\bar{v}_m, c-(m-2)\Delta c\}$$
$$- \min\{\bar{v}_m, c-(m-1)\Delta c\}][1 - F_m(\min\{\bar{v}_m, c-(m-2)\Delta c\})] \prod_{i=1}^{m-1} (1 - F_i(\Delta c))$$

(where we define $F_0(\Delta c) = 0$) which is negative. This implies that for each $\omega \in \Omega$, expression (3) is negative. Therefore, expression (2) is negative. Q.E.D.

Given that a classical ex post efficient, weakly incentive feasible allocation mechanism does not exist, we examine other appealing mechanisms.

5. An auction-like mechanism

The auction-like mechanism is appealing because it is simple, strongly incentive feasible, and universal. The mechanism is universal in the sense that neither the participants nor the mechanism designer need to have any statistical information regarding others' preferences.

The simple mechanism is an auction-like mechanism (alm), which works as follows. A coordinator (i.e., an auctioneer) announces a price vector $p = (p_0, p_1,...,p_n)$, where

$p_m = c(n, m)/m$. (The price p_m the equal cost share should m individuals consume the good.) Each individual must announce one m, whose interpretation is that he will accept to contribute as long as there are at least m contributors (including himself). Let ω_{p_m} be the set of agents that announce m or higher. We have $\omega_{p_{m'}} \subseteq \omega_{p_m}$ if $m' \leq m$. The coordinator chooses set ω_{p_m} with the greatest m.

We now define the direct mechanism that corresponds to this simple auction-like mechanism. To do this, we define the set

$$\omega^{\mathrm{alm}}(v) = \max_{\omega \in \Omega} \#\left|\omega\colon v_i - \frac{c(n, m_\omega)}{m_\omega} \geq 0 \text{ for each } i \in \omega\right|.$$

(Note that solution to this optimization program may be $\omega^{\mathrm{alm}}(v) = \emptyset$. In this case, the collective action is not taken.)

For the auction-like mechanism, the provision rule is defined by

$$\rho_\omega^{\mathrm{alm}}(v) = \begin{cases} 1 & \text{if } \omega = \omega^{\mathrm{alm}}(v), \\ 0 & \text{if } \omega \neq \omega^{\mathrm{alm}}(v). \end{cases}$$

The net monetary transfer rule is defined by

$$\tau_{i_\omega}^{alm}(v) = \begin{cases} c(n, m_{\omega^{\mathrm{alm}}(v)})/m_{\omega^{\mathrm{alm}}(v)} & \text{if } \omega = \omega^{\mathrm{alm}}(v) \text{ and if } i \in \omega^{\mathrm{alm}}(v), \\ 0 & \text{if } \omega \neq \omega^{\mathrm{alm}}(v). \end{cases}$$

Lemma 4. *The auction-like mechanism,* $(\rho^{\mathrm{alm}}(v), \tau^{\mathrm{alm}}(v))$, *is strongly incentive feasible.*

Proof. Let v be some vector of reports and let $i \in \omega^{\mathrm{alm}}(v)$. Then, i's payoff from any report $\hat{v}_i \geq v_i$ is $v_i - c(n, m_{\omega^{\mathrm{alm}}(v)})/m_{\omega^{\mathrm{alm}}(v)} \geq 0$; and i's payoff from any report $\hat{v}_i < v_i$ is no greater than $v_i - c(n, m_{\omega^{\mathrm{alm}}(v)})/m_{\omega^{\mathrm{alm}}(v)}$. Let $i \not\in \omega^{\mathrm{alm}}(v)$. Then, i's payoff from any report $\hat{v}_i < v_i$ is zero; and i's payoff from any report $\hat{v}_i$ is non-positive. Therefore the auction-like mechanism is dominant strategy incentive compatible and ex post individually rational. The ex post balanced budget requirement follows directly from the definition of the mechanism. Q.E.D.

We now turn to the case of n individuals allocating a fixed cost of an excludable public good. That is, the cost function for this case is $c(n, m) = x$, for $x > 0$. Theorem 2 establishes that the auction-like mechanism approaches classical ex post efficiency (because the probability each individual in the economy consumes the public good approaches 1) as n approaches infinity.

Theorem 2. *Consider the collective action problem* $(n, c(n, m), (F^1,...,F^Q), (\beta^1,...,\beta^Q))$, *where the cost function is* $c(n, m) = x$, *for* $x > 0$. *Using the auction-like mechanism—*$(\rho^{\mathrm{alm}}(v), \tau^{\mathrm{alm}}(v))$*—to allocate the public good, the probability that individual i (for each* $i \in N$*) consumes the public good approaches 1 as n approaches infinity.*

Proof. Consider the distribution F^q, $q \in \{1,...Q\}$. According to Serfling (1980, p. 77, Theorem A), the distribution of the pth-order statistic, ξ_{pn}, of the sample $(v_1,...,v_{\beta^{q}n})$ (where each v_i is drawn according to the distribution function F^q) is asymptotically normal and its variance approaches zero as n approaches infinity. Thus, there exists m' such that for each $m > m'$,

$$\mathrm{prob}\left(\# |v_i \geq \frac{x}{m}| \geq m \right) \rightarrow 1 \text{ as } n \rightarrow \infty.$$

Therefore, the probability of consumption with at least m (for $m > m'$) individuals approaches 1 as n goes to infinity. Then as $m \rightarrow \infty$, the probability that individual i consumes the public good, $E_v \sum_{\omega \in \Omega} g_\omega(i) \rho^{\mathrm{alm(n)}}(v)$, approaches 1. Q.E.D.

6. Ex ante optimality and the α* mechanism

While the auction-like mechanism has several appealing properties, it is not generally efficiency. Thus, we now turn to the examination of ex ante optimal mechanisms. One particular class of mechanisms (ρ, τ) characterizes ex ante optimal mechanisms. This class involves the specification of virtual valuations.[8] To determine virtual valuations, fix some non-negative scalar α. Individual i's virtual valuation of the collective action is defined by

$$\psi_i(v_i, \alpha) = v_i - \alpha \frac{1 - F_i(v_i)}{f_i(v_i)}.$$

We now define α* mechanisms—the class of mechanisms that characterize the ex ante optimal mechanisms. To do this, we define the set

$$\omega(v, \alpha) = \arg\max_{\omega \in \Omega} \sum_{i \in \omega} \psi_i(v_i, \alpha) - c(n, m_\omega)$$

subject to

$$\sum_{i \in \omega} \psi_i(v_i, \alpha) - c(n, m_\omega) \geq 0$$

and

$$\psi_i(v_i, \alpha) - [c(n, m_\omega) - c(n, m_{\omega - 1})] \geq 0 \text{ for each } i \in \omega.$$

(Note that the solution to this optimization program may be $\omega(v, \alpha) = \emptyset$. In this case, the collective action is not taken.)

The α schedule associated with an α mechanism

$$\rho^\alpha(v) = (\rho^\alpha_{\omega_1}(v), \ldots, \rho^\alpha_{\omega_{2^n}}(v))$$

is defined by

$$\rho^\alpha_\omega(v) = \begin{cases} 1 & \text{if } \omega = \omega(v, \alpha) \\ 0 & \text{if } \omega \neq \omega(v, \alpha). \end{cases}$$

[8] Virtual valuations are part of the characterization of many ex ante optimal mechanisms for incomplete information mechanism design problems. See Myerson (1981); Myerson and Satterthwaite (1983); Gresik and Satterthwaite (1989), and Mailath and Postlewaite (1990a) among others.

After the transformation of valuation v_i to virtual valuation $\psi_i(v_i, \alpha)$, the α schedule $\rho^{\alpha}(v)$ states that, if the collective action is taken, individual i is included in the provision if and only if i's virtual valuation ψ_i is no less than the cost of provision $c(n, m_{\omega(v,\alpha)}) - c(n, m_{\omega(v,\alpha)} - 1)$. A nonempty consumption group, $\omega(v, \alpha)$, exists if and only if the sum of virtual valuations of those who consume the collective action is no less than the total cost of the provision. Therefore, in terms of virtual valuations, the α-schedule, $\rho^{\alpha}(v)$, of the ex ante optimal mechanism satisfies a standard economic efficiency requirement.

Substitution of $\rho_{\alpha}(v)$ into (2) for the n-individual economy yields

$$G_n(\alpha) = \int \cdots \int \times \sum_{\omega \in \Omega} \rho^{\alpha}_{\omega}(v) \left(\sum_{i \in N} g_{\omega}(i) \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) - c(n, m_{\omega}) \right) \prod\nolimits_k f_k(v_k) \, \mathrm{d}v_k \geq 0.$$

Let $\alpha^* \equiv \min\{\alpha \in (0, 1): G_n(\alpha) \geq 0\}$. (A direct implication of Theorem 1 is that $\alpha > 0$. If $\alpha = 1$, then the collective action is taken if and only if the virtual surplus is positive, and thus $G_n(\alpha = 1) > 0$. Hence, $\alpha^* \in (O, 1)$.)

An α schedule $\rho^{\alpha}(v)$ is an α^* schedule if and only if $\alpha = \alpha^*$ and $\Sigma_{\omega \in \Omega}\, g_{\omega}(i) \tilde{\rho}_i^{\alpha}(\cdot)$ is nondecreasing over $[\underline{v}_i, \bar{v}_i]$ for each $i \in$N. By definition, an α^* schedule satisfies Lemma 2's requirements. Therefore, should an α^* schedule exist, then a payment schedule $\tau^{\alpha *}$ exists such that the allocation mechanism $(\rho^{\alpha *}, \tau^{\alpha *})$ is weakly incentive feasible. We call this mechanism $(\rho^{\alpha *}, \tau^{\alpha *})$ the α^* mechanism for the collective action problem $(n, c(n, m), (F_1,...,F_n))$.

Lemma 5 establishes the existence of an α^* mechanism for the collective action problem $(n, c(n, m), (F_1,...,F_n))$. The α^* mechanism is ex ante optimal among the class of weakly incentive feasible mechanisms for $(n, c(n, m), (F_1,...,F_n))$.

Lemma 5. *For each collective action problem* $(n, c(n, m_{\omega}), (F_1,...,F_n))$, *if* $\omega_i(v_i, 1)$ *is increasing for each* $i \in N$, *the* α^* *mechanism* $(\rho^{\alpha *}, \tau^{\alpha *})$ *exists, is weakly incentive feasible, maximizes the expected surplus among the class of weakly incentive feasible mechanisms, and has positive expected gains from agreement.*

Proof. The proof of Lemma 5 is in Appendix A.

We now consider one particular class of collective actions—excludable public goods. We examine the case in which n individuals consider sharing the fixed cost, $x > 0$, of constructing a public good. In Lemma 6, we examine how the efficiency of the α^* mechanism is related to the size of the economy (n). We demonstrate that $\alpha^*(n') < \alpha^*(n)$ for each $n' > n$. In this sense, the ''inefficiency α-wedge,'' used to determine virtual valuations from actual valuations is decreasing in the size of the economy.

Lemma 6. *Consider the collective action problems* $(n, c(n, m), (F_1,...,F_n))$ *and* $(n', c(n', m), (F_1,...,F_{n'}))$, *where* $N \subset N'$ *and the cost function is* $c(n, m) = c(n', m) = x$, *for* $x > 0$. *Then,* $\alpha^*(n') < \alpha^*(n)$ *and moreover* $\omega_n(v, \alpha^*(n)) \subseteq'_{n'}(v, \alpha^*(n'))$.

Proof. For $\alpha^*(n)$, $G_n(\alpha^*(n))=0$. Then for $n'>n$,

$$G_{n'}(\alpha^*(n)) = G_n(\alpha^*(n)) + \left\{ \sum_{i=n+1}^{n'} \int_{\underline{v}_i}^{\bar{v}_i} \sum_{\omega\in\Omega} \rho_\omega^{\alpha^*(n)}(v) g_\omega(i)\left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)}\right) f_i(v_i)\,\mathrm{d}v_i \right\}.$$

The bracketed term { } is positive. Thus, $G_{n'}(\alpha^*(n))>0$ and $\alpha^*(n')<\alpha^*(n)$.

Next, fix a valuation vector, v, for an initial group $N'=\{1,...,n'\}$. Consider the initial group of size n, where $n<n'$, and label the individuals $i=1,...n$. For the consumption group $\omega_n(v, \alpha^*(n))$, $\Sigma_{i\in\omega(v,\alpha^*(n))}\psi_i(v_i,\alpha^*(n))\geq x$, and for each $i\in\omega_a(v, \alpha^*(n))$, $\psi_i(v_i,\alpha^*(n))\geq 0$. Next, fix $\alpha^*(n)$ and consider the consumption group $\omega_{a'}(v, \alpha^*(n))$. For $\omega_{n'}(v, \alpha^*(n))$, $\Sigma_{i\in\omega'(v,\alpha^*(n))}\psi_i(v_i,\alpha(n))\geq x$, and for each $i\in\omega_{a'}(v, \alpha^*(n))$, $\psi_i(v_i,\alpha^*(n))\geq 0$. Thus, $\omega_n(v, \alpha^*(n))\subseteq\omega_{n'}(v, \alpha^*(n'))$. Q.E.D.

For the allocation of a fixed cost, Theorem 3 establishes that the α^* mechanism approaches classical ex post efficiency as the economy becomes very large.

Theorem 3. *Consider the collective action problem* $(n, c(n, m), (F^1,...,F^Q), (\beta^1,...,\beta^Q))$, *where the cost function is* $c(n, m)=x$, *for* $x>0$. *For the* α^* *mechanism,* $\alpha^*(n)\to 0$ *as* $n\to\infty$.

Proof. The proof follows directly from Theorem 2. If the auction-like mechanism approaches classical ex post efficiency as n approaches infinity, the α^* mechanism also approaches classical ex post efficiency. Thus, $\alpha^*(n)\to 0$ as $n\to\infty$. Q.E.D.

For the allocation of a fixed cost, $x>0$, the auction-like mechanism is very attractive in that it is simple, strongly incentive feasible for each n, and for large groups (i.e. as n approaches infinity) it approaches classical ex post efficiency. For smaller groups and for cases in which weak incentive feasibility is an appropriate feasibility requirement, there is a welfare loss of the auction-like mechanism compared to the α^* mechanism. To investigate the size of this welfare loss, we numerically compare the expected gains of the auction-like mechanism to those of the α^* mechanism. We examine a two-individual economy in which each agent's value is uniformly distributed on the unit interval, and vary the provision cost, x.

Table 1 reports the results. In this table, the total ex ante expected gains to trade are denoted by $T(x)$, and the ex ante gains to trade with the auction-like mechanism are

Table 1
Properties of the Ex-Ante Optimal and Auction-Like Mechanisms for a Two-Individual Economy

$c(2, m)=x$	$T(x)$	α^*	$T^{\alpha^*}(x)$	$T^{\text{alm}}(x)$	$T^{\text{alm}}(x)/T^{\alpha^*}(x)$
0.0	1.000000	0.000000	1.000000	1.000000	1.000000
0.2	0.810333	0.086081	0.794433	0.793000	0.998196
0.4	0.610666	0.158068	0.591224	0.584000	0.987781
0.6	0.436000	0.226160	0.405852	0.391000	0.963405
0.8	0.285333	0.292278	0.251314	0.232000	0.923148
1.0	0.166667	0.333333	0.140625	0.125000	0.888889

denoted by $T^{\mathrm{a/m}}(x)$ and with the α^* mechanism are denoted by $T^{\alpha*}(x)$. The last column shows that the relative efficiency of the auction-like mechanism to the α^* mechanism, $T^{\mathrm{a/m}}(x)/T^{\alpha*}(x)$, is decreasing in x. Also, note that the inefficiency wedge, α^*, is increasing in x.

7. The ex ante optimal limiting mechanism and Ramsey pricing

In this section we characterize the ex ante optimal mechanism for very large groups (i.e., for a continuum of individuals, $n=\infty$). We return to the general case of collective actions, where $c(n, m)$ can depend on both n and m. We examine the following type of step-function mechanism:

$$\rho_\omega(v) = \begin{cases} 1 & \text{if } \omega = \tilde{\omega}, \\ 0 & \text{otherwise;} \end{cases} \tag{7}$$

and

$$\tau_{i_\omega} = \begin{cases} \tilde{v}_i & \text{if } v_i \geq \tilde{v}_i \text{ and } \omega = \tilde{\omega}, \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $\tilde{\omega} = \{i: v_i \geq \tilde{v}_i\}$.In this mechanism, individual i consumes the good if and only if his valuation is no less than the critical value $\tilde{v}_i$, and he pays $\tilde{v}_i$ to consume the good.

We consider the collective action problem $(n, c(n, m), (F^1,...,F^Q), (\beta^1,...,\beta^Q))$ and determine the step-function mechanism (ρ, τ) that maximizes the expected surplus per individual subject to the ex post balanced budget constraint. We need some notation. Let m^q denote the number of individuals from group q who consume the good, and let

$$S(m) = \left\{(m^1,...,m^Q): \sum_{q=1}^{Q} m^q = m\right\}.$$

The expected cost associated with this step-function mechanism is

$$E[c(n, m)] = \sum_{m=1}^{n} \sum_{(m^1,\dots,m^Q)\in S(m)} \prod_q \binom{\beta^q n}{m^q} (1 - F^q(\tilde{v}^q))^{m^q} (F^q(\tilde{v}^q))^{\beta^q n - m^q} c(n, m).$$

For the step-function mechanism, the constrained optimization program for maximizing the per capita gains to agreement subject to the ex ante balanced budget condition is

$$\begin{aligned} &\max_{\tilde{v}^q,\ q=1,\dots,Q,\ \lambda} \sum_q \beta^q \int_{\tilde{v}^q}^{\bar{v}^q} v_i f^q(v_i)\, \mathrm{d}v_i - \sum_{m=1}^{n} \sum_{(m^1,\dots,m^Q)\in S(m)} \prod_q \binom{\beta^q n}{m^q} \\ &(1 - F^q(\tilde{v}^q))^{m^q} (F^q(\tilde{v}^q))^{\beta^q n - m^q} \frac{c(n, m)}{n} \\ &+ \lambda \Bigg[\sum_q \beta^q (1 - F^q(\tilde{v}^q)) \tilde{v}^q - \sum_{m=1}^{n} \sum_{(m^1,\dots,m^Q)\in S(m)} \prod_q \binom{\beta^q n}{m^q} \\ &(1 - F^q(\tilde{v}^q))^{m^q} (F^q(\tilde{v}^q))^{\beta^q n - m^q} \frac{c(n, m)}{n} \Bigg]. \end{aligned} \tag{9}$$

Let y^q denote the value of $F^q(\tilde{v}^q)$. The first-order conditions for the constrained optimization program yield

$$\frac{\tilde{v}^{q*} - \dfrac{\partial E[c(n,m)]}{\partial y^q}}{\tilde{v}^{q*}} \leq \frac{\lambda^*}{1+\lambda^*} \frac{1-F^q(\tilde{v}^{q*})}{f^q(\tilde{v}^{q*})\tilde{v}^q} \tag{10}$$

with equality if $\tilde{v}^{q*} < \bar{v}^q$ for each $q = 1, \ldots, Q$.

The equations in (10) are precisely Ramsey pricing.[9] Let $\tilde{v}^* = (\tilde{v}^1, \ldots, \tilde{v}^{Q*})$ denote the vector of Ramsey prices. The Ramsey-pricing mechanism is defined by the step-function mechanism, (7) and (8), where $\tilde{v}_i = \tilde{v}^{q*}$ if $F_i(v_i) = F_q(v_i)$. For the n-individual collective action problem, the Ramsey-pricing mechanism is denoted by $(\rho^{\tilde{v}*(n)}, \tau^{\tilde{v}*(n)})$. To interpret this as Ramsey pricing, the l.h.s. term of (10)

$$\frac{\tilde{v}^{q*} - \dfrac{\partial E[c(n,m)]}{\partial y^q}}{\tilde{v}^{q*}}$$

is the Lerner index. The numerator of this term is the price to segment q less the marginal expected cost associated with raising the value of $\tilde{v}^q$. Next, consider the r.h.s. of (10). The expected demand by a type-q individual is $(1 - F^q(\tilde{v}^{q*}))$. The term

$$\frac{1-F^q(\tilde{v}^{q*})}{f^q(\tilde{v}^{q*})\tilde{v}^{q*}}$$

on the r.h.s. of (6) is then the reciprocal of price elasticity. Therefore, Eq. (6) is Ramsey pricing: the Lerner index equals the reciprocal of price elasticity. We demonstrate in Theorem 4 that for a continuum of individuals Ramsey pricing is ex ante optimal.

Theorem 4. *Consider a continuum of individuals* (i.e., $n = \infty$). *For the collective action problem* $(\infty, \mathrm{ac}^\infty(\gamma), (F^1, \ldots, F^Q), (\beta^1, \ldots, \beta^Q))$*, the ex-ante optimal mechanism,* $(\rho^{\alpha*(\infty)}, \tau^{\alpha*(\infty)})$*, is deterministic and is characterized by the Ramsey-pricing mechanism* $(\rho^{\tilde{v}*(\infty)}, \tau^{\tilde{v}*(\infty)})$*. Moreover, the limiting mechanism is strongly incentive feasible.*

Proof. We demonstrate that for $n = \infty$, the optimal mechanism is deterministic. Consider the distribution function $F^q(v_i)$ $(q \in \{1, \ldots, Q\})$. According to (Serfling, 1980, p. 77, Theorem A), the distribution of the pth-order statistic, $\tilde{\xi}_{pn}$, of the sample $(v_1, \ldots, v_\beta)$ (where in this case each v_i is drawn according to the distribution function F^q) is asymptotically normal and the variance of the distribution approaches zero as n approaches infinity. Thus in the limit, the realized distribution of types from the class-q population coincides with the prior distribution, described by F^q. This means that the $\alpha^*(\infty)$ mechanism is a step-function mechanism that maximizes the expected gains to agreement subject to the weak incentive feasibility constraint. That is, the $\alpha^*(\infty)$ mechanism is a Ramsey-pricing mechanism.

[9]See Laffont and Tirole (1993), for more one Ramsey pricing and incomplete information problems.

The proof that the mechanism is strongly incentive feasible is straightforward. First, from the definition of the step-function mechanism ((7) and (8)) it is clearly ex post individually rational. Second, because individual i's choice is effectively binary—consume at price $\tilde{v}_i$ or do not consume at price zero—individual i has a dominant strategy to honestly report v_i. Third, because the realized distribution of types coincides with the prior distribution of types, ex ante individual rationality coincides to ex post individual rationality. Ex post individual rationality is therefore the constraint in the maximization program (9). Q.E.D.

Remark. If the cost function is continuous, then a sequence $\rho^{\tilde{v}(n)}$ exists such that $\rho^{\tilde{v}(n)} \to \rho^{\alpha *(\infty)} = \rho^{\tilde{v} *(\infty)}$ as $n \to \infty$. (Note that the sequence may not specify Ramsey prices.)

8. Discussion

R-M-P examine negotiation mechanisms that allocate the cost of a pure public good (i.e., exclusion is infeasible) and demonstrate that if a mechanism is weakly incentive feasible, then the probability the collective action is taken goes to zero as the group size becomes very large. In this paper, we showed that the possible exclusion from the benefits of a collective action improves the efficiency of cost allocation negotiations and is part of an information-constrained Pareto efficiency allocation mechanism. This result is contrary to the popular belief that such exclusion is Pareto inefficient: Individuals who value the collective action more than the marginal cost should be able to consume the good. Of course, this belief is based on the presumption that individual valuations are public information and this belief thus ignores the demand-revelation issues.

One important point to recognize about exclusion is that once the collective action has been funded and the inclusion set (ω) is determined, no additional people are permitted to take part in the collective action. However, for each incentive feasible mechanism, there are some individuals who do not consume the good (i.e., are not in ω), but are willing to pay their marginal costs. After the set ω is determined, these excluded people may offer to renegotiate the mechanism by offering a price slightly greater than marginal cost in order to consume the good. This renegotiation would constitute an ex post Pareto improving move. However, any type of renegotiation must be part of a mechanism. As we demonstrated in this paper, any mechanism, even one that incorporates such renegotiation, cannot ex ante dominate an α^* mechanism. Thus, ex ante, there are no gains to designing a mechanism that incorporates renegotiation. (Renegotiation may improve the efficiency of the auction-like mechanism because this mechanism is not ex ante efficient.)

Renegotiation may be prevented. For example, the owners of professional sports teams do not open the gates for free admission once the game begins and the stadium is not full. The incentive problems of doing so are clear: if individuals expect free entry at game time, they wait until then to enter the stadium. The owners must establish a reputation that they will not allow free entry at game time. Our exclusion mechanism must rely on the same type of reputation to enforce the exclusion once the collective action has been funded.[10]

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Appendix A

Lemma 4. *For each collective action problem* $(n, c(n, m), (F_1, \ldots, F_a))$, *if* $\psi_i(v_i, 1)$ *is increasing for each* $i \in N$, *the* α*-mechanism $(\rho^{\alpha*}, \tau^{\alpha*})$ exists, is weakly incentive feasible, maximizes the sum of expected surplus among the class of weakly incentive feasible mechanisms, and has positive expected gains from agreement.*

Proof. The constrained optimization program for ex ante optimality is

$$\max\,(1+\lambda)\int\cdots\int\sum_{\omega\in\Omega}\rho_\omega(v)$$
$$\left(\sum_{i\in N}\left(g_\omega(i)\left(v_i-\frac{\lambda}{1+\lambda}\,\frac{1-F_i(v_i)}{f_i(v_i)}\right)-c(n, m_\omega)\right)\right)\prod_k f_k(v_k)\,\mathrm{d}v_k$$

The function is maximized by $\rho^\alpha(v)$ when $\alpha=\lambda/(1+\lambda)$. The provision rule $\rho^\alpha(v)$ satisfies the constraint (2) to the optimization program with equality. It is important to note that because the cost function is not convex in m, other provision rules may satisfy the constraint (2) with equality. Therefore, the first-order conditions to the maximization program are not sufficient for a global maximum. But, among the class of provision rules that satisfy (2), $\rho^\alpha(v)$ specifies the largest group $\omega(v, \alpha)$ to consume the public good and $\rho^\alpha(v)$ is non-decreasing in v.To demonstrate existence of a solution, we show that $G_n(\alpha)$ is increasing and continuous. We first establish that $\alpha^*\in(0, 1)$. By Theorem 1, $G_n(0)<0$. Because $\Sigma_{i\in N}\bar{v}_i>c(n, n)$, $G_n(1)>0$. We now demonstrate that $G_n(\alpha)$ is increasing. Suppose $\delta>\alpha$. For every v and ω such that $\rho^\alpha_\omega(v)\neq\rho^\delta_\omega(v)$, $\rho^\alpha_\omega(v)=1$ and $\rho^\delta_\omega(v)=0$. Moreover,

[10] See Dearden (1991), for an analysis of reputation-building that prevents the renegotiation of classical ex post Pareto inefficient allocation mechanisms.

$$\left(\sum_{i\in\omega(\alpha)}\left(v_i-\alpha\,\frac{1-F_i(v_i)}{f_i(v_i)}\right)-c(n,m_{\omega(\alpha)})\right)<0$$

implies

$$\left(\sum_{i\in\omega(\alpha)}\left(v_i-\frac{1-F_i(v_i)}{f_i(v_i)}\right)-c(n,m_{\omega(\alpha)})\right)<0$$

because $\alpha^*<1$. This implies that $G_n(\delta)\geq G_n(\alpha)$. That is, $G_n(\cdot)$ is increasing.Now, we demonstrate the continuity of $G_n(\alpha)$. For an α-mechanism, let $D_\omega(\alpha)$ represent the set of valuations for which the group ω consumes the public good. That is, $D_\omega(\alpha)=\{v\colon \rho^\alpha_\omega(v)=1\}$. We define $D_\omega(\alpha,\delta)=\{v\colon \rho^\alpha_\omega(v)=1$ and $\rho^\delta_\omega(v)=0\}$. We demonstrate that for each $\omega\neq\emptyset$, $\Pr[D_\omega(\alpha,\delta)]\to 0$ as $|\delta-\alpha|\to 0$. Because the function $\psi_i(v_i,1)$ is strictly increasing in v_i, the function $\psi_i(v_i,\alpha)$ is strictly increasing in v_i for any $\alpha>0$. Hence, given ω, v_{-i} and α,

$$\sum_{i\in\omega}\psi_i(v_i,\alpha)-c(n,m_\omega)=0$$

has at most one solution in v_i denoted by $\hat{v}_i(v_{-i},\alpha)$, and the solution varies continuously in v_{-i} and α. Also, given ω, v_{-i}, and α,

$$\psi_i(v_i,\alpha)-[c(n,m_\omega)-c(n,m_\omega-1)]=0$$

has at most one solution in v_i, denoted by $\tilde{v}_i(v_{-i},\alpha)$, and the solution varies continuously in v_{-i} and α. The value $v'_i(v_{-i},\alpha)=\max\{\hat{v}_i(v_{-i},\alpha),\tilde{v}_i(v_{-i},\alpha)\}$ is the smallest value of v_i given (v_{-i},α), for which individual i is included in the collective action. The value $v'_i(v_{-i},\alpha)$ also varies continuously in v_{-i} and α. From the continuity of $v'_i(v_{-i},\alpha)$, then $\Pr[D_\omega(\alpha,\delta)]\to 0$ as $|\delta-\alpha|\to 0$. Therefore,

$$\begin{aligned}G_n(\delta)-G_n(\alpha)=\sum_{\omega\in\Omega}\int\cdot\cdot\\
\cdot\int_{D_\omega(\alpha,\delta)}\Bigg(\Bigg[\rho^\delta_\omega(v)\Bigg(\sum_{i\in N}g_\omega(i)\psi_i(v_i,1)-c(n,m_\omega)\Bigg)\\
-\rho^\alpha_\omega(v)\Bigg(\sum_{i\in N}g_\omega(i)\psi_i(v_i,1)-c(n,m_\omega)\Bigg)\Bigg]\Bigg)\prod\nolimits_{k\in\omega}f_k(v_k)\,\mathrm{d}v_k\to 0\end{aligned}$$

as $|\delta-\alpha|\to 0$ because $\Pr[D_\omega(\alpha,\delta)]\to 0$ as $|\delta-\alpha|\to 0$. From Theorem 1, $G(0)<0$. Thus, $\alpha^*\in(0,1)$ exists such that $G_n(\alpha^*)=0$. Q.E.D.

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