Strategic Manipulation of University Rankings, the Prestige Effect, and Student University Choice

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ABSTRACT

A multiperiod, theoretical model characterizes the relationship between a publication that ranks universities and prospective students who might use this ranking to decide which university to attend. The published ranking offers information about the objective quality of universities but also affects their prestige, which may increase student utility. This prestige effect gives the commercial publication incentive to act contrary to the best interest of the students. If a ranking created with the commonly used attribute-and-aggregate methodology creates prestige, then to maximize profit the publication needs to (1) choose attribute score weights that do not match student preferences and (2) alter those attribute score weights over time, even in the absence of changes to student preferences or education technology. Without a prestige effect, the publication should choose attribute score weights that match student preferences. This model also defines a student-optimal ranking methodology that maximizes the sum of the utilities of students. The results offer insights for prospective students who use existing rankings to choose a university, as well as which ranking designs would better align with students' preferences.

According to the College Board, the annual average tuition for public universities in the 2017–18

school year was $9,970 for in-state and $25,620 for out-of-state students; for private universities,

it has reached $34,740 (Ma, Baum, Pender, and Welch 2017). Such vast expenses, combined

with the complexity of the university service offering, makes the decision of which college to

attend a difficult and challenging one (Avery and Levin 2010). Students and their families (i.e.,

consumers) often seek information to make these decisions from publicly available rankings of

universities (Griffith and Rask 2007; Luca and Smith 2009),[1] including the *U.S. News & World*

Report (*USNWR*) and *The Wall Street Journal/The Times Higher Education* rankings of U.S.

colleges and universities, *BusinessWeek* and its ranking of U.S. MBA programs, *The Times*

Higher Education ranking of international universities, and *Maclean's* ranking of Canadian

universities. The proliferation of such ranking guides attests to their importance for various

university stakeholders beyond students, including alumni, donors, administrators, and faculty.

The rankings influence student decisions, but students (and other stakeholders) may be naïve

regarding the strategic goals of commercial ranking publications (driven by profit rather than

public service motives) and their effects on how the publications produce rankings.

      Commercial college and university ranking publications receive frequent criticisms over

the methodologies they use; sentiment about university rankings in general and *USNWR* in

particular is so strong that an entire Wikipedia page is devoted solely to criticisms of college and

university rankings (Wikipedia Authors, August 22, 2018). A common complaint is that the

publications "tinker" with their ranking methods from year to year. As Tierney (2013) writes in

The Atlantic:

> *U.S. News* is always tinkering with the metrics they use, so meaningful
> comparisons from one year to the next are hard to make. Critics also allege that

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[1] Expert advice, including product rankings, has proven important in markets outside of higher education including
health care (Pope 2009), consumer products (Simonsohn 2011), and consumer services (Jin and Leslie 2003).

> **this is as much a marketing move** [emphasis added] as an attempt to improve the quality of the rankings: changes in the metrics yield slight changes in the rank orders, which induces people to buy the latest rankings to see what's changed.

That is, changes to the ranking methodology may have more to do with the marketing of these publications than with improving the quality of the ranking. With this research, we seek to identify the features of the market for college education that might induce reputable publications, like *USNWR*, to modify its ranking methodology continually, and even in a static setting with no changes in universities to select a methodology that interjects uncertainty into the outcome of its ranking. Given that these rankings have been around for so long, it is surprising that such modifications are so persistent and that the rankings may create volatility among students.

All university stakeholders together constitute the target audience for commercial rankings, yet we focus specifically on students here because their choice of university and the consequences of that choice are central. We seek to identify factors that might induce a publication to manipulate its ranking methodology for business purposes; we do not pursue a comprehensive explanation of the decisions that rankings publications make regarding their methodologies. Thus, we analyze a market comprised of one target audience and a monopoly ranking publication. We construct a multiperiod theoretical model of how information (university rankings) gets transmitted strategically by an expert (the publication) to less informed decision makers (students).

In our model, the publication's ranking – the rank-order of the universities – provides information to students, who are uncertain about universities; and the ranking creates prestige, which students desire, for highly ranked universities.[2] The methodology used by the publisher to

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[2] The ordering of universities provides information about their quality, whether students interpret the ranks as salient indicators of overall quality or use those ranks to update (probabilistic) beliefs about the universities' performance on specific attributes. A published ranking for a university may confer prestige on that university, which we model as the increase in the utility a student experiences when the chosen university improves its ranking. In our model,

form its ranking is implicit in our student decision model of whether to view the ranking and which university to attend, and we characterize a publication's optimal ranking methodology relative to two extreme versions: a *viewing-student optimal ranking methodology*, which uses student preferences to rank universities, and a *uniform/random ranking methodology*, which uses a uniform distribution over all possible rankings of universities and essentially selects a ranking at random. The former is optimal for the students who view the ranking and use it in their decision-making process; the latter creates the greatest possible uncertainty about the outcome. Our analysis of this model reveals that three factors—(1) the commercial publication's profit incentive, (2) the ability of the ranking publication to alter the prestige granted to universities, and (3) asymmetric information between the publication and students about the quality of universities—combine to provide incentives for a publisher to create ranking methodologies that are best for the publisher but not best for students.

Our analysis provides three main insights. First, the prestige effect of university rankings provides an incentive for a commercial publication to use a ranking methodology that does not match student preferences, pushing the publication away from a viewing-student optimal ranking methodology and toward a uniform/random ranking methodology. Second, from a dynamic perspective, when a prestige effect grows over time, the publication optimally begins with the methodology that is best for the students who view the ranking but has an incentive to move away over time, toward one that adds randomness to its rankings. Third, when a prestige effect is present, the publication selects a ranking methodology that adds more uncertainty to the ranking than is optimal, both for students who view the ranking and for those who do not view it.

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ranked universities may be status goods, which Grossman and Shapiro (1984, p. 82) define as "goods for which the mere use or display of a particular branded product confers prestige on their owners, apart from any utility deriving from their function."

In the next section, we provide a brief history of university ranking publications, and how the rankings work; further, we relate our analysis to research on advertising, fashion cycles, and media bias, which have institutional features similar to those of the university ranking marketplace. Then we provide the formal set up of our model and present our general results about the publisher's optimal ranking methodology within a particular period and over time. We also investigate an all-student-optimal methodology (which maximizes the sum of utilities of students who view and do not view the rankings), and demonstrate how it differs from both the viewing-student optimal and the publication's profit-maximizing methodologies. Finally, in discussing our results and limitations, we consider how students might cope with the current information environment and propose modifications to university ranking methodologies that might mitigate the incentives for the strategic manipulation of rankings.

PRODUCT RANKINGS AND RELATED LITERATURE

Rankings

University rankings first appeared in the 1870s to inform higher education scholars, professionals, and government officials (Stuart 1995), but they gained mass appeal in 1983 when *USNWR*, using a survey of university presidents, published its first rankings of undergraduate academic quality. It included the top-25 national universities and top-25 national colleges. In 1987, *USNWR* adopted its current multidimensional ranking methodology, incorporating more objective attributes and assessments by academic leaders of peer institutions; in the following year, it expanded its rankings of the national universities to the top-50 universities. By 2004, *USNWR* had created three categories (national doctoral universities, regional master's universities, and national colleges), and then its 2015 edition launched its best global universities, ranked both overall and by subject, with separate rankings for Asia and Latin

America. In addition, it ranks graduate schools, by 11 specialties, and high schools. The 2019 *USNWR* ranking involves attribute categories that include assessments by administrators at peer institutions, student retention, faculty resources, student selectivity, financial resources, alumni giving, and graduation performance.

Students use these publications to acquire information that affects their decision processes; a better rank in an influential rankings publication leads to significant increases in applications (Luca and Smith 2009) and matriculation rates (Griffith and Rask 2007). University administrators, despite some criticisms, recognize these rankings as publicly visible performance scorecards and voluntarily participate in the system, by providing data about attributes that are of interest to students, as (potential) customers. Hobart and William Smith College fired a senior vice president in 2000 after she failed to submit fresh data to *USNWR*, which led to a major drop in the College's rank (Graham and Thompson 2001). Richard Beeman, Dean of the College of Arts and Sciences at the University of Pennsylvania, commented in a letter to *The New York Times* (September 17, 2002), "I breathed a sigh of relief when my university continued to appear in the [*USNWR*] top 10."

The most common method among commercial publishers to rank universities is the *attribute-and-aggregate* methodology in which a publication identifies key university attributes (e.g., classes with fewer than 20 students, acceptance rate), rates each university on those attributes, chooses a weight for each attribute (a publisher's key decision variable), multiplies the weight by the rating, and adds the weighted sum of attribute ratings to form a university score. The publication then rank-orders universities based on these scores to form a ranking.

Publications in fields other than education use attribute-and-aggregate ranking methodologies (e.g., *Consumer Reports*, *Cook's Illustrated* for cooking equipment, the tire retailer Tire Rack).[3]

Attribute-and-aggregate undergraduate university rankings might rely on four classes of university attributes: (1) preferences or utilities of entering students (measured, for example, by the universities they choose to attend), (2) student inputs to the university (e.g., quality of the entering first-year class), (3) quality of the student experience (e.g., retention, university resources, alumni giving, graduation rates), and (4) the universities' economic value (e.g., net tuition, graduate school and employment placement of university graduates, salaries following graduation, loan repayment rates). We know of no university ranking that includes all four attribute classes. Whereas *USNWR* relies on attributes of the entering class and measures of student experiences, *The Wall Street Journal/The Times Higher Education* ranking of U.S. undergraduate programs includes information about student experiences (e.g., student engagement, teaching quality) and economic value (e.g., average net tuition, average annual salaries 10 years after graduation).

In determining a methodology, each publication that ranks universities pursues the primary goal of garnering revenue by generating traffic, through the sales of the magazine or access to the website, which in turn may increase advertising revenues. To generate student traffic, the rankings must provide value to students, which exists only if students expect the information to influence their decisions about which universities to apply to and attend. In turn, four strategic decisions by rankings publications can affect the value they provide students:

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[3] As an alternative to the attribute-and-aggregate approach, product scores could be based on aggregated customer satisfaction levels or votes, as in *The New York Times*'s bestseller list and *USA Today's* Coaches' *Poll* of college football teams. Dai et al. (2014) propose an algorithm to aggregate consumer reviews on Yelp.com that weights reviews according to their perceived value; in an interesting variation of survey- or voting-based methodologies, Avery et al. (2013) derive a revealed preference ranking of U.S. colleges and universities using students' matriculation decisions.

prices for access and advertising, attribute scores included in the rankings, investments in measuring and collecting attribute scores, and the functions used to aggregate attribute scores into a ranking. Our analysis focuses primarily on the publication's strategic choice of an aggregation function—that is, the weights it uses to aggregate attribute score weights into overall scores, which is equivalent to a choice of ranking methodology. We note that *USNWR* regularly changes its methodology; on its "Frequently Asked Questions: 2019 Best College Rankings" page, *USNWR* even cites the question, "Why does the methodology change most years?"[4]

Advertising, Fashion Cycles, and Media Bias

Rankings likely are consumed in a manner similar to advertising (or other communications), in that they provide information about universities and create prestige for highly ranked ones, similar to how advertisements provide information and create status for brands. The information and prestige perspectives parallel research in advertising, such that some studies regard advertising as an information source, but others refer to it as a persuasion tool (Bagwell 2007). From an information perspective, rankings inform consumers (students, families) about product (university) attributes (Butters 1977; Nelson 1974; Stigler 1961). From a persuasion perspective, rankings influence the prestige associated with adopting and consuming the product (university attended) (Ackerberg 2001; Becker and Murphy 1993; Galbraith 1976).

This dichotomy (information versus persuasion) also applies to product positioning, which is at least partially the result of advertising. Information-oriented advertising encourages product positioning on functional dimensions while persuasive messages prompt positioning on meaning and signaling dimensions (Fournier 1998). Consider luxury goods, such as designer watches. Firms that advertise them normally stress exclusivity and the prestige they convey

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[4] http://www.usnews.com/education/best-colleges/articles/rankings-faq; accessed October 15, 2018.

rather than the functionality of the item (i.e., a basic ability to keep accurate time; Han, Nunes, and Drèze 2010). Students' selection of a university in turn has some characteristics in common with luxury buyers' consumption decisions for watches. If future employers use university ranks to assess the human capital of university graduates, students have an economic reason to attend better-ranked universities, independent of the quality of the education they receive. In addition, students obtain internal satisfaction and bragging rights by attending a highly ranked school. Just as advertising can affect the utility a consumer derives from consuming the advertised product, a university's rank can affect the utility a student obtains from attending the university.

We also draw on literature on fashion cycles and media bias. Publications of university rankings have an incentive to inject randomness into the evaluation process, just as fashion magazines do for the "it" items they select (e.g., Kuksov and Wang 2013). Furthermore, news organizations have incentives to bias their news coverage to appeal to viewers with particular political preferences, just as university rankings publications manipulate their ranking methodologies to generate interest among certain groups of prospective students (Zhu and Dukes 2015). To delineate these links and our reliance on the fashion cycle and media bias literature further, we first elaborate on our model and discuss its link to these two research streams in more detail in the discussion section.

MODEL DEVELOPMENT

With the proposed model, we primarily seek insight into factors that might cause a for-profit publication to select a university ranking methodology that is not optimal for students and to change its ranking methodology strategically over time, even without any changes in student preferences or educational technology. By comparing its profit-maximizing ranking

methodology against one that is best for all types of students, we also demonstrate that these same factors cause the publication to interject excessive uncertainty into its ranking.[5]

In our model, the ranking publication functions as a profit-maximizing expert that achieves its profit goals through revenue maximization; revenues, whether from access, advertising, or the sale of detailed university information, increase with more views of its ranking. Expertise emerges from the publication's attempts to learn universities' attribute scores before ranking them. Therefore, in our model in which the publication knows the relative importance of each attribute to students, it can rank universities in a manner consistent with student utility, whether it actually does so or not.

The students, unlike the publication, are novices; they have probabilistic beliefs about attribute scores, and they choose whether to use the expert advice (i.e., view the ranking) before deciding which university to attend. The university ranking provides them with information in two main ways. First, as incorporated in our model, it gives students information about attribute scores. Second, not incorporated into our model, a publication may have expertise in evaluating the relative importance that students should assign to various attributes.

Our model also characterizes universities by their attribute scores, such as the average SAT scores of the entering classes and student-to-faculty ratios. Because universities do not take any actions in our model, we do not treat them as actors but rather as the focal institutions that students attend and that the publication ranks.

Timing

During each period t of our multiperiod model, the information states and order of events are as follows:

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[5] In the Web Appendix, we construct and analyze a stylized example with two universities and two attributes to illustrate this point.

(1) Students engaged in university search processes and the publication are both uncertain about the universities' period t attribute scores but have probabilistic beliefs about them. Students know the number of period t-1 students who have viewed period t-1 rankings.

(2) The publication, which uses an attribute-and-aggregate ranking methodology, chooses the weights in its aggregation function that it attaches to attribute scores.[6]

(3) The publication learns the attribute scores and ranks the universities.

(4) Students choose whether to view the ranking. Those who do view the ranking update their probabilistic beliefs about the attribute scores.

(5) Students choose which universities to attend.

Universities

Our model contains n universities, each operating in every period $t = 1, 2, \ldots$ University $i, i \in N = \{1, \ldots, n\}$, in period t is characterized by m attribute scores, $a_i^t = (a_{i1}^t, \ldots, a_{im}^t)$. Let $a^t = (a_1^t, \ldots, a_n^t)$ denote a profile of the attribute scores of all universities. At the beginning of period t, each student and the publication develop probabilistic beliefs. Let $p_{ij}^t(a_{ij}^t)$ represent the probability density function for university i's jth attribute score a_{ij}^t, $p_i^t(a_i^t)$ indicate the probability density function for a_i^t, and $p^t(a^t)$ be the probability density function for a^t. The probability density functions allow, for example, the publication and students to believe that a flagship state university is likely to have a higher student-to-faculty ratio than a small private university. In general, students and the publication expect universities to vary by attributes.

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[6] The publication sets its ranking methodology before learning attribute scores. Furthermore, it reports neither its ranking methodology nor the universities' attribute scores; in practice, publications typically offer only an outline of their methodologies and (possibly for a fee) an incomplete list or approximate attribute scores. In our model, the publication does not report its period t ranking methodology to the students, but students can infer its equilibrium ranking methodology, because they know the publication's objective function.

Furthermore, the probability density functions may change over time, reflecting potential changes in the distribution of a university's expected quality.

The Publisher and Rankings

In each period, the publication uses an attribute-and-aggregate ranking methodology in which it attaches weights to attribute scores, determines each university's overall score by calculating the weighted sum of its attribute scores, and ranks universities according to the overall scores. We primarily focus on an optimal, linear, attribute-and-aggregate methodology, but in the Web Appendix, we also define and analyze a general ranking methodology.

A ranking in period t is denoted r^t, and universities are ranked from the best in position 1 through the worst in position n. University i's position in the period t ranking r^t is r_i^t, and the set of all possible rankings is $\mathcal{R}$. In an attribute-and-aggregate ranking methodology, university i's period t aggregated score is a weighted sum of its attribute scores, $\sum_{j=1}^{m} w_j^t a_{ij}^t$, where w_j^t denotes the weight that the publication assigns to attribute j in the period. The publication ranks universities according to the aggregated scores: If $\sum_{j=1}^{m} w_j^t a_{ij}^t > \sum_{j=1}^{m} w_j^t a_{i'j}^t$, then $r_i^t < r_{i'}^t$.[7]

With profit maximization as its strategic goal, in each period of its dynamic problem, the publication designs a ranking methodology. Because our focus is the design of the ranking methodology, not the pricing of click-through advertisements or subscriptions, we can assume a simple, linear relationship between the number of views of a ranking and the revenues generated by that ranking. In this linear relationship, revenue per view is constant, so if the publication generates revenues by click-through ads, the rate per click-through is constant across the number of clicks. If the publication generates revenues by selling detailed information about colleges and

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[7] Despite the loss of generality in assuming a linear ranking methodology, it does permit convex preferences. For example, using our notation, attribute scores could be the natural log of academic spending per student and the average SAT score of an entering class. If so, the iso-aggregate score surface would be strictly convex in academic spending per student and average SAT scores.

universities or its content in general, the demand for such information is assumed to be perfectly price elastic. In addition, the costs of publishing an online ranking are almost entirely fixed, associated with developing the ranking methodology, collecting the necessary data, and designing the presentation of the online content. Costs that vary with the number of views of the ranking are negligible, so we set the marginal cost to the publication of the number of views to 0.

The number of students s^t who view the publication's period t ranking is a function of the period t ranking methodology, $w^t = (w_1^t, \ldots, w_m^t)$, as well as the number of students who have viewed the ranking in period $t - 1$, s^{t-1}. The publication's period t revenue generated from its ranking publication thus is the product of the revenue generated per student who views the ranking x, whether x is the price of the ranking or the revenue generated per student who views it, and the number of students who view the period t ranking s^t. The publication's revenue over all periods is $\sum_{t=1,2,\ldots} x \, s^t$. Its objective is to choose weights w^t in each period t to maximize the number of students who view the ranking (equivalent to maximizing revenue):

(1) $\quad max_{w^t, t=1,2,\ldots} \sum_t x s^t(w^t; s^{t-1})$.

The function $s^t(w^t; s^{t-1})$ is akin to a demand function in which the number of students who view the ranking depends on the publication's choice of w^t, and s^{t-1} is a parameter.[8] We derive this demand function by characterizing students' decisions to view the ranking.

Students

Students make two decisions: whether to pay the price (in terms of either time or money) to view a ranking and which university to attend. The expected utility of attending a university depends on the utility derived from the university's attribute scores, the utility derived from the

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[8] Because students do not observe w^t, their choices about whether to view the ranking are a function of their belief about w^t, which in equilibrium is consistent with w^t.

university's rank, the student's probabilistic beliefs about the university's attribute scores

(updated if the student views the ranking, and not otherwise), and the student's probabilistic

beliefs about the university's rank (learned if the student views the ranking). A student selects

the university that offers the greatest expected utility and chooses to view the ranking if and only

if the increase in expected utility from doing so is greater than the cost in money or time. In each

period t, our model depicts a new unit-mass batch of students that enters the education market,

where all universities admit all student applicants and students attend universities for only one

period.[9]

Student Utility. A period t student is concerned with the attributes of the universities a^t

and the university ranks r^t only in period t. The attribute scores reflect the quality of the

education and student experience available at the universities. The ranks have the potential to

create prestige; students and their families gain pride and satisfaction from attending top-ranked

universities and sharing that information with others. Furthermore, employers use universities'

ranks as information about the quality of the graduating students and compete to recruit students

from better-ranked universities. If the prestige of the universities' ranks is important to students,

the actual ranks may affect the utilities of not just students who view the rankings but also those

who do not view them: those non-viewing students may still obtain objective benefits, through

increased employment opportunities, and psychological benefits by eventually learning of the

rank sometime in the future.

A student determines the utility of attending a university by assigning a weight to each

individual attribute score, a weight to the aggregated attribute score, and a weight to the

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[9] If the universities are selective (i.e. do not admit all students) and the prestige effect is present, our general result that the for-profit publication would add randomness to the viewing-student optimal ranking methodology would continue to hold. However, the publication would choose a ranking methodology that would add less variance to the viewing-student optimal one.

university's rank. Let γ_j^t be the weight a student in period t assigns to attribute j and α^t be the weight the student assigns to the aggregated attribute score. The component of a student's utility from attending university i that includes the attribute scores is $\alpha^t \sum_j \gamma_j^t a_{ij}^t$.

The prestige component is the product of three terms. First, we denote the importance of the prestige of attending university i to the student by the weight β^t. Second, prestige is important only if people view (and are knowledgeable about) the ranking. Thus, the importance of a ranking's prestige on students is a function of the ranking's impact, gauged by its popularity. This impact $g(s^{t-1})$ is a function of the number of views in the immediate past period, which accounts for a multiplier effect associated with views of the publication's ranking, associated with word-of-mouth advertising about the ranking, on the magnitude of the prestige effect.[10] Third, a student's utility for attending a university is strictly decreasing in the university's rank (the top university is ranked 1 and not n). The strictly decreasing function $\rho(r_i^t)$ captures the effect of university i's rank on utility. With these three components, the effect of the prestige of attending university i on a student's utility is $\beta^t g(s^{t-1}) \rho(r_i^t)$. [11] Thus, the student's overall utility from attending university i is:

(2) $U_i^t = \alpha^t \sum_j \gamma_j^t a_{ij}^t + \beta^t\, g(s^{t-1})\rho(r_i^t).$

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[10] This specification is consistent with word-of-mouth communication and sales research (Chevalier and Mayzlin 2006; Peres, Muller, and Mahajan 2010). The student's utility in reality may depend on both the university's current rank and its ranks in past or even potential future ranks. The magnitude of the prestige created by a ranking also may depend on the number of people who view the ranking over these periods. This multiperiod influence of rankings on current student utility yields the same two basic results about the prestige effect, in that it creates an incentive for a publication to choose attribute score weights that do not match student preferences and change those attribute score weights strategically over time.

[11] The prestige component of the student's utility function, $\beta^t\, g(s^{t-1})\, \rho(r_i^t)$, is a mathematical shortcut. Regarding future careers, a student's utility from attending a university should be a function of the job s/he intends to obtain and the willingness of employers to make job offers. In addition, the bragging rights a student obtains from attending a highly ranked university should be a function of social interactions that generate status. In specifying our model, we bypass the analysis of the labor market and social interactions by depicting the student's utility for attending a university directly as a function of the university's rank.

Students differ in the weights they assign to the information provided by the ranking α^t and the prestige of the rankings β^t in their utility functions. The values of α^t and β^t for the period t student population are distributed according to the population distribution functions $H_\alpha^t(\alpha^t)$ and $H_\beta^t(\beta^t)$, with corresponding density functions $h_\alpha^t(\alpha^t)$ and $h_\beta^t(\beta^t)$. Although students differ in the importance they attach to attribute scores overall and to university ranks, we assume all students attach the same weights to specific attribute scores in their utility functions, $\gamma^t = (\gamma_1^t, \ldots, \gamma_m^t)$, so we can examine whether the publication chooses to rank universities in a manner consistent with the students' preferences. The publication knows $\gamma^t = (\gamma_1^t, \ldots, \gamma_m^t)$, $H_\alpha^t(\alpha^t)$, and $H_\beta^t(\beta^t)$.

Student Beliefs. Students view the ranking to infer universities' objective quality and learn about the universities' prestige. Those who do not view the ranking do not learn about the attribute scores (do not update probabilistic beliefs) or the prestige created by university rank. However, they form probabilistic conjectures about the ranks, consistent with the equilibrium weights the publication used to rank the universities. That is, prior to deciding whether to view the ranking, all period t students hold probabilistic beliefs $p^t(a^t)$ about attribute scores. Depending on the publication's equilibrium ranking methodology w^t and students' prior beliefs about attribute scores $p^t(a^t)$, students form probabilistic beliefs about rankings. Those who do not view the ranking believe that $q^t(r^t|a^t; w^t)$ is the probability of the ranking r^t, conditional on the attribute scores a^t (which the publication knows) and the publication's ranking methodology w^t. Because they do not observe the actual attribute scores, the probability distribution over rankings that is relevant to them must account for their uncertainty about a^t. From $q^t(r^t|a^t; w^t)$ and $p^t(a^t)$, we can determine the probability distribution over rankings r^t that is relevant to their decision to view the ranking:

(3) $\quad q^t(r^t; w^t) = \int_{a_{11}^t} \cdots \int_{a_{nm}^t} q^t(r^t|a^t; w^t)\, p^t(a^t) da_{11}^t \cdots da_{nm}^t.$

Students who do not view the ranking continue to hold the probabilistic beliefs expressed by Equation 3 when they choose which university to attend.

Before making this choice, students who view the ranking update their probabilistic beliefs about attribute scores according to the ranking they observe and the publication's equilibrium ranking methodology, using Bayes rule. The probability of a^t, conditional on r^t and w^t, is $p^t(a^t|r^t; w^t)$. As we noted, students who choose not to view the ranking continue to hold the prior probabilistic beliefs about attribute scores, $p^t(a^t)$.

Student Expected Utility and Decision Rules. If a student views the ranking and learns r^t, the expected utility of attending university i is:

(4) $\quad \begin{aligned} E[U_{i,View}^t(w^t; s^{t-1})] &= \alpha^t \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t|r^t; w^t) \sum_j \gamma_j^t a_{ij}^t\, da_{11}^t \cdots da_{nm}^t \\ &\quad + \beta^t g(s^{t-1})\rho(r_i^t). \end{aligned}$

This expected utility is the weighted sum of two components: the expected weighted sum of university i's attribute scores, for which the student uses the updated probability distribution of attribute scores $p^t(a^t|r^t; w^t)$, and the prestige effect of university i's rank. Given the ranking r^t, the student chooses to attend the university with the greatest expected utility, as expressed in Equation 4. Therefore, to calculate the expected utility of viewing the ranking for each possible r^t, the student determines the university with the greatest expected utility of attending. The student then uses the equilibrium probability distribution over ranks, $q^t(r^t; w^t)$ as specified in Equation 3, to calculate the expected utility of viewing the ranking:

(5) $\quad V_{View}^t(w^t; s^{t-1}) = \sum_{r^t \in \mathcal{R}} q^t(r^t; w^t) max_i E[U_{i,View}^t(w^t; s^{t-1})] - c,$

where c denotes the student's cost of viewing the ranking, which is either the purchase price of the ranking or the cost of the student's time needed to view the ranking.

If a student chooses not to view the ranking, the expected utility of attending university i is:

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(6) \quad E\big[U^t_{i,Not}(w^t; s^{t-1})\big] = \int_{a^t_{11}} \cdots \int_{a^t_{nm}} \big(\alpha^t p^t(a^t) \sum_j \gamma^t a^t_{ij}
$$
$$
+ \beta^t \sum_{r^t \in \mathcal{R}} q^t (r^t | a^t; w^t) g(s^{t-1}) \rho(r^t_i)\big) da^t_{11} \cdots da^t_{nm}.
$$

This expected utility is the weighted sum of two components as well: the expected weighted sum of university i's attribute scores, for which the student uses the prior probability distribution of attribute scores $p^t(a^t)$, plus the expected prestige effect of university i's rank, for which s/he uses the publication's equilibrium conditional probability of rankings, $q^t(r^t | a^t; w^t)$. The student then calculates the expected utility of attending each university and attends the university with the greatest expected utility:

$$
(7) \quad V^t_{Not}(w^t; s^{t-1}) = max_i E\big[U^t_{i,Not}(w^t; s^{t-1})\big].
$$

A student chooses to view the publication if and only if $V^t_{View}(w^t; s^{t-1}) - c \geq V^t_{Not}(w^t; s^{t-1})$. We use the function,

$$
(8) \quad I^t(w^t; s^{t-1}) = \begin{cases} 1 & if \; V^t_{View}(w^t; s^{t-1}) - c \geq V^t_{Not}(w^t; s^{t-1}); \\ 0 & otherwise \end{cases}
$$

to indicate whether a student in period t views the publication. The number of students who view the period t publication is:

$$
(9) \quad s^t(w^t; s^{t-1}) = \int \int I^t(w^t; s^{t-1}) h^t_\alpha(\alpha^t) h^t_\beta (\beta^t) d\alpha^t d\beta^t.
$$

We can draw a line $V^t_{View}(w^t; s^{t-1}) - c = V^t_{Not}(w^t; s^{t-1})$ to partition the (α^t, β^t) space into students who view the ranking $((\alpha^t, \beta^t)$ above the line) and those who do not $((\alpha^t, \beta^t)$ below the line). The function $s^t(w^t; s^{t-1})$ is the proportion of students who view the ranking (i.e., in the (α^t, β^t) space with values of (α^t, β^t) above the line).

Equilibrium

The equilibrium that we examine is a perfect Bayesian equilibrium (PBE), which requires

that the students' choices (universities to attend and whether to view the ranking) and the

publication's choice of ranking methodology are sequentially rational. A PBE also requires that

students' probabilistic beliefs be consistent with equilibrium play.

Similar to the problem we examine, cheap talk games can have multiple PBE, so we turn

to that domain for insight. Some cheap talk analyses construct refinements of the PBE concept to

generate unique equilibria (e.g., Chen, Kartik, and Sobel 2008; Crawford and Sobel 1982); others

focus on one of the PBE (e.g., Che and Kartik 2009). Our model has multiple equilibria and a

continuum of PBE, such that for each w^t, there is a PBE in period t that yields w^t as an

outcome.[12] We focus on the equilibrium in which the publication chooses its ranking

methodologies $w^t, t = 1, 2, ...$, which maximizes the number of students who view the ranking,

$\sum_{t,t=1,2,...} s^t(w^t; s^{t-1})$.

PUBLICATION'S PROFIT-MAXIMIZING RANKING METHODOLOGY

In this section, we demonstrate that the prestige effect provides an economic incentive for

the publication to set attribute score weights that differ from students' weights and adjust those

weights over time, simply to induce more students to view them. We begin by establishing that

to identify the publication's optimal ranking methodology for each period in the dynamic model,

we only need to consider the number of views in $t-1$, not the future equilibrium between the

publication and students (Lemma 1). Then we identify sufficient conditions for which the

publication's optimal ranking methodology in period t is best for viewing students or

uniform/random (Lemma 2, Theorem 1). By examining the dynamics, we establish that in certain

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[12] The model has multiple equilibria because for each student profile regarding who decides to view the ranking, the publisher is indifferent among all values of w^t.

stability conditions, the number of students who view the ranking grows over time (Lemma 3); with the growing popularity of the ranking, its prestige effect increases as well. Finally, with no previous views, the publication begins in period 1 with the viewing-student optimal ranking methodology. Then, as the number of students who view the ranking increases and the prestige effect grows accordingly, the publication moves its ranking methodology farther from the viewing-student optimal version and closer to a uniform/random approach (Theorem 2).

The following preliminary result simplifies our analysis (for proofs, see Appendix A):

Lemma 1 If $\left(w^{1^*}, w^{2^*}, \dots\right) = argmax \sum_t xs^t(w^t; s^{t-1})$, then for each t, $w^{t^*} = argmax\ xs^t(w^t; s^{t-1})$.

In our dynamic model, we can calculate the optimal period t ranking methodology without looking ahead. The publication takes the number of views from $t - 1$ and sets w^t to maximize s^t without regard to the effect of w^t on future views, $s^{t+1}, s^{t+2}, \dots$. The intuition stems from recognizing that the number of future views increases with the number of current views.

Methodology for Period t

In analyzing the period t ranking methodology, we show that the prestige effect provides an incentive for a publication to use attribute score weights that do not match student weights and to rank universities in a manner that is inconsistent with student preferences. We consider two special cases: no students experience a prestige effect; or all students experience only a prestige effect, without any influence of university attributes. Through these two cases, we establish bounds on the publisher's profit-maximizing ranking methodology and demonstrate that in any period t of our dynamic model, under these extreme conditions, the optimal ranking methodology is either viewing-student optimal or uniform/random ranking. A *viewing-student optimal ranking methodology* indicates the publication uses student preferences to determine

each university's score and therefore ranks the universities according to $w^t = \gamma^t$. The

uniform/random ranking methodology instead relies on a uniform distribution to randomly select

the positions of the schools in the ranking, such that for each r^t, $q^t(r^t) = \frac{1}{n!}$.[13] We use Lemma 2

in the proof of Theorem 1.

Lemma 2 Consider period t.

(a) If students are unconcerned with prestige ($H_\alpha^t(0) < 1$ and $H_\beta^t(0) = 1$) or the period $t - 1$

ranking has no views ($s^{t-1} = 0$), when it chooses its ranking methodology to maximize

$s^t(w^t; s^{t-1})$, the publication maximizes $V_{View}(w^t; s^{t-1})$.

(b) If students are concerned with only prestige ($H_\alpha^t(0) = 1$ and $H_\beta^t(0) < 1$) and the ranking

in period $t - 1$ ranking has views ($s^{t-1} > 0$), when it chooses its ranking methodology to

maximize $s^t(w^t; s^{t-1})$, the publication minimizes $V_{Not}(w^t; s^{t-1})$.

Theorem 1

(a) If students are unconcerned with prestige ($H_\alpha^t(0) < 1$ and $H_\beta^t(0) = 1$) or the period $t - 1$

ranking has no views ($s^{t-1} = 0$), the viewing-student optimal ranking methodology ($w^{t^*} = $

γ^t) maximizes s^t.

(b) If students are concerned with only prestige ($H_\alpha^t(0) = 1$, $H_\beta^t(0) < 1$) and the ranking in

period $t - 1$ ranking has views ($s^{t-1} > 0$), a uniform/random ranking methodology,

$q_R(r^t) = \frac{1}{n!}$, maximizes s^t.

---

[13] It is possible that no linear attribute-and-aggregate ranking methodology is uniform/random, in which case the publication could use an alternative ranking methodology. In general, a ranking methodology is a conditional probability density function, $q^t(r^t|a^t)$; for each possible profile of attribute scores a^t, it specifies the probability q^t of ranking r^t, conditional on the profile. For example, one uniform ranking methodology involves "babbling," such that the publication chooses each ranking with equal probability, regardless of the actual attribute scores, $q^t(r^t) = \frac{1}{n!}$, for each a^t. In the Web Appendix, we demonstrate that Theorem 1 holds in a model with general utility functions in which the publisher is not restricted to using an attribute-and-aggregate ranking methodology.

That is, the publication's strategic goal to maximize its profit, combined with students' concern about prestige, causes the publication to add randomness to its ranking. If all students are concerned only with prestige, and unconcerned with information about attribute scores, the publication instead selects a ranking methodology that makes each possible ranking of the universities equally likely. In contrast, if all students are concerned only with information about attribute scores provided (probabilistically) by the ranking, not with prestige, the publication chooses a ranking methodology that matches the students' preferences.

To clarify this result, consider the effects of the ranking on students' expected utility from viewing the ranking and not viewing the ranking. Each student wants to attend the top-ranked school. If a student views the publication, she can choose the top university; if she does not view the publication, she may make a mistake and attend a lower ranked university. To maximize the net utility of viewing the ranking, the publication seeks to maximize the probability that a student who does not view the published ranking makes a university selection mistake. It does so by putting each university in each position with equal probability.

Alternatively, if students are concerned only with the attribute scores and the ranking does not create prestige, this ranking does not affect the expected utilities for students who do not view the ranking. Therefore, to maximize the number of students who view the ranking, the publication needs to maximize students' expected utilities from viewing the rankings. It does so by choosing a ranking methodology that matches student preferences.

Theorem 1 thus offers insight into two special cases, but it does not address the more realistic cases in which students are concerned with both information and prestige. We examine these cases in a multiperiod context.

Dynamics

If students are unconcerned with prestige, the publication's chosen viewing-student optimal ranking methodology remains unchanged. Any shifts in the rankings over time would depend only on changes in the universities' own attribute scores. Consider students who are concerned with both university attribute scores and prestige though. The publication still begins with no established ranking, so its period 1 ranking creates no prestige. Without a prestige effect in period 1, the publication strategically selects a viewing-student optimal ranking methodology. In period 2, first-period views have occurred, so the ranking gains a prestige effect. In turn, the optimal ranking methodology moves away from the viewing-student optimal one and toward a uniform/random one. As the number of students who view the ranking continues to increase over time, the publication moves in each subsequent period, away from the viewing-student optimal and toward the uniform/random ranking methodology.

We investigate a stable environment, in which the distributions of the student utility weights α and β and the universities' attribute scores remain unchanged over time. The number of students who view the ranking grows over time, and so does the prestige effect of the ranking.

Lemma 3 If for each t' and t'', students have the same weights for each attribute score ($\gamma^{t'} = \gamma^{t''}$), the weights $(\alpha^{t'}, \beta^{t'})$ and $(\alpha^{t''}, \beta^{t''})$ are identically distributed ($H_\alpha^{t'} = H_\alpha^{t''}$ and $H_\beta^{t'} = H_\beta^{t''}$), and the attribute scores are identically distributed ($p^{t'} = p^{t''}$), then for any t, $s^t > s^{t-1}$.

If the popularity s^t of the publisher's university ranking changes over time, the relative importance to students of the prestige effect changes as well. In response, the publisher changes its profit-maximizing ranking methodology.

Because depicting changes for general n-university, m-attribute cases is complicated and offers no additional insights beyond the two-university, two-attribute case, we examine a simple 2×2 case, in which student utility weights α and β are uniformly distributed on $[0, \bar{\alpha}]$ and $[0, \bar{\beta}]$ respectively, with $\bar{\alpha}$ and $\bar{\beta}$ satisfying an additional assumption specified in the theorem regarding the cutoff in the (α, β) space between students who view the ranking and students who do not. We justify the assumption of uniform distributions for α^t and β^t and the cutoff assumption prior to the proof of Theorem 2 in Appendix A. In our analysis of this two-attribute case, without loss of generality, for each t, we can normalize $\gamma_2^t = 1$ and $w_2^t = 1$. Finally, for each t, we set $\gamma_1^t = \gamma_1$. With $w_2^t = 1$, in each period t, the publication chooses only w_1^t; we let $w_1^{uniform}$ denote the weight that implements a uniform/random ranking methodology. Then, we define $\hat{\alpha}(\beta^t ; w^t, s^{t-1})$ as the value of α^t as a function of β^t for which $V_{View}^t - V_{Not}^t = c$. The $\hat{\alpha}$ function is the cutoff in the (α^t, β^t) space between the students who the view the ranking and the students who do not.

Theorem 2 Consider a case of two universities, $n = 2$, that have two attributes, $m = 2$. In each period t, the student weights α^t and β^t are uniformly distributed, and $H_\alpha(\alpha^1) = H_\alpha(\alpha^2) = \cdots = \frac{\alpha^t}{\bar{\alpha}}$, and $H_\beta(\beta^1) = H_\beta(\beta^2) = \cdots = \frac{\beta^t}{\bar{\beta}}$, respectively. In addition, $\bar{\alpha}$ is sufficiently large such that for each $t, t = 1, 2, ...,$ and for each $\beta^t \in [0, \bar{\beta}]$, $\hat{\alpha}(\beta^t ; w^t, s^{t-1}) \in [0, \bar{\alpha}]$. If $\gamma_1 > w_1^{uniform}$, then for each t and $t - 1$, $w_1^{t^*} < w_1^{t-1^*} < w_1^{1^*} = \gamma_1$; if instead $\gamma_1 < w_1^{uniform}$, then for each t and $t - 1$, $w_1^{t^*} > w_1^{t-1^*} > w_1^{1^*} = \gamma_1$.

As the publication grows in popularity over time and the effect of its prestige on student utility increases, the attribute score weight w_1^t moves away from the viewing-student optimal and toward the uniform/random ranking methodology. The publication's objective is to interject

more uncertainty over time about its actual ranking and thus increase the number of students who view it.

THE WELFARE OF ALL STUDENTS

We have established that the prestige effect is a key determinant of a commercial publisher's selection of a ranking methodology that is not best for the students who view the ranking. In this section, we also consider students who do not view the ranking to determine whether a change in the publisher's ranking methodology, away from a profit-maximizing form, could benefit all students.

When there is no prestige effect, students who do not view the ranking are indifferent to the methodology the publication uses (because it does not affect their utilities), and the publication uses a methodology that is best for students who view the ranking. That is, without a prestige effect, the selected ranking methodology is best for all students. However, in the presence of a prestige effect, all students are affected by the publication's choice to maximize profit by interjecting uncertainty into its ranking methodology. For students who view the ranking, the profit-maximizing methodology adds uncertainty to the viewing-student-optimal ranking methodology. For students who do not view the ranking, they gain utility if their preferred university (which gives them the greatest expected utility based on the attribute scores) is more likely to be ranked first. Therefore, all students would be better off if the ranking methodology shifts away from a profit-maximizing and toward a viewing-student optimal one. Dynamically, as the ranking gains views over time and the prestige effect grows, the publication may add uncertainty to its ranking methodology, though all students would prefer less uncertainty. Lemma 4 examines the optimal ranking methodology for students who do not view the ranking.

Lemma 4 Consider period t. Suppose $s^{t-1} > 0$.

(a) For a non-viewing student unconcerned with prestige ($\beta^t = 0$), the expected utility of not viewing the ranking $V_{Not}^t(w^t; s^{t-1})$ is constant in the probability that the publication selects ranking r^t, $q^t(r^t; w^t)$, and is therefore constant in w^t.

(b) For a non-viewing student concerned with prestige ($\beta^t > 0$), if university i is the ex ante preferred university, such that

(10) $\quad \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j \gamma_j^t a_{ij}^t \, da_{11}^t \cdots da_{nm}^t >$

$\quad max_{i',i' \in N \setminus i} \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j \gamma_j^t a_{i'j}^t \, da_{11}^t \cdots da_{nm}^t,$

the expected utility from not viewing the ranking $V_{Not}^t(w^t; s^{t-1})$ is maximized if the publication ranks university i first.

Next we examine two universities and two uniformly distributed attribute scores. For each t, we normalize $\gamma_2^t = 1$ and $w_2^t = 1$. With $w_2^t = 1$, in each period t, the publication chooses only w_1^t.

Theorem 3 Consider a case of two universities, $n = 2$, with two attributes, $m = 2$, in period t, with $s^{t-1} > 0$. In each period t, the student weights α^t and β^t are uniformly distributed on $[0,1]$. Assume without loss of generality that $q^t((1,2); w_1^t)$ is strictly increasing in w_1^t.

(a) If all students are unconcerned with prestige ($H_\beta^t(0) = 1$), $V_{View}^t(w_1^t; s^{t-1})$ is maximized with the publication's profit-maximizing ranking methodology, $w_1^{t^*} = \gamma_1^t$, and $V_{Not}^t(w_1^t; s^{t-1})$ is constant in w_1^t. A change in w_1^t from the publication's profit-maximizing ranking methodology would make non-viewing students no better off and viewing students worse off.

(b) If all students are concerned with prestige ($H_\beta^t(0) = 0$), and $q^t((1,2); w_1^{t^*}) > 1/2$, then $V_{View}^t(w_1^{t^*}; s^{t-1})$ and $V_{Not}^t(w_1^{t^*}; s^{t-1})$ are strictly increasing in w_1^t at the publication's

profit-maximizing ranking methodology, $w_1^{t^*} < \gamma_1^t$. An increase in w_1^t at $w_1^{t^*}$ would make all students better off. If $q^t\left((1,2); w_1^{t^*}\right) < 1/2$, then $V_{View}^t(w_1^{t^*}; s^{t-1})$ and $V_{Not}^t(w_1^{t^*}; s^{t-1})$ are strictly decreasing in w_1^t at the publication's profit-maximizing ranking methodology, $w_1^{t^*} > \gamma_1^t$. A decrease in w_1^t at $w_1^{t^*}$ would make all students better off.

As Theorem 3 demonstrates, the prestige effect, combined with the publication's expertise and strategic goal to maximize profit, causes the publication to select a ranking methodology that all students agree could be improved. Students agree that the publication interjects uncertainty into its ranking, and as Lemma 3 indicates, students who do not view the ranking want the publication to rank their preferred university first with a probability of 1. Furthermore, students who view the ranking prefer that the publication use $w_1^t = \gamma_1^t$. Because the publication sets $w_1^{t^*}$ in between γ_1^t and $w_1^{uniform}$, students who view the ranking also perceive that the publication interjects uncertainty into its ranking.

If we examine the summed utilities of all students, the publication's profit-maximizing ranking methodology interjects more uncertainty into its ranking than is best for students collectively. An *all-student optimal ranking methodology* would maximize the sum of student utilities:

(11) $\quad \int \int \left(\left(V_{\{View\}}(w^t; s^{t-1}) - c\right)I^t(w^t; s^{t-1}) + V_{\{Not\}}(w^t; s^{t-1})(1 - I^t(w^t; s^{t-1}))\right),$

which we denote as $w^{t^{aso}}$.

Corollary 1 Consider a case of two universities, $n = 2$, with two attributes, $m = 2$, in period t, with $s^{t-1} > 0$. Assume without loss of generality that $q^t((1,2); w_1^t)$ is strictly increasing in w_1^t.

(a) If all students are unconcerned with the prestige of university ranks ($H_\beta^t(0) = 1$), the publication's profit-maximizing ranking methodology is all-student optimal, $w_1^{t^*} = w_1^{t^{aso}} = \gamma_1^t$.

(b) If some or all students are concerned with prestige ($H_\beta^t(0) < 1$), and students who do not view the ranking attend university 1 (in Equation 10, $i = 1$), then $w_1^{t^*} < w_1^{t^{aso}}$ and $q^t((1,2); w_1^{t^{aso}}) > q^t((1,2); w_1^{t^*}) > \frac{1}{2}$. If instead students who do not view the ranking attend university 2 (in Equation 10, $i = 2$), then $w_1^{t^*} > w_1^{t^{aso}}$ and $q^t\left((1,2); w_1^{t^{aso}}\right) < q^t\left((1,2); w_1^{t^*}\right) < \frac{1}{2}$.

We turn to the question of whether students are better off due to the publication's ranking depends on whether the ranking has a prestige effect. If the ranking provides information but creates no prestige, it has no effect on the utilities of students who do not view the ranking, but it increases the utilities of students who view it. Therefore, introducing a ranking without any prestige effect is a Pareto improvement. However, if the ranking has a prestige effect, some students are made better off but others are worse off due to the ranking. For example, if top employers use the ranking to identify universities from which they will recruit, students who view the ranking and attend the top universities benefit, but students who do not view the ranking may not attend these universities and thus not be considered by top employers. Without the ranking, top employers would spread their interview resources across universities.

CONCLUSION

Summary of Findings

The extent to which the methodology a publication uses to rank universities aligns with student preferences depends on whether the publication creates prestige for universities. If a

university ranking has no prestige effect, the publication's optimal ranking methodology matches student preferences. If it does have a prestige effect though, the publication's optimal ranking methodology diverges from student preferences, and the publication has an incentive to change its ranking methodology over time, due to the difference between the profit motive of the publication and the utility function of students. If students were to place more weight on the informative role of rankings and less weight on the prestige of attending highly ranked universities, the publication would move its profit-maximizing ranking methodology closer to students' preferences. However, it may be optimal for students to include a prestige effect term in their utility functions if, for example, the employment market for college graduates uses university rankings as signals of their quality.

When a prestige effect is present, all students can be made better off by moving away from a profit-maximizing ranking methodology and toward a one with less variability. Students who view the ranking prefer that the publisher use the viewing-student optimal ranking methodology to reduce uncertainty about the methodology it uses, while it seeks to maximize its profit. Students who do not view the ranking prefer that the publisher eliminate all uncertainty and select their ex ante preferred university.

Link with Fashion Cycle and Media Bias Literature

A prominent university ranking can create prestige for high-ranked universities; an influential fashion magazine similarly can create prestige for people who wear the items that it identifies as stylish. According to Kuksov and Wang (2013, p. 53), fashion editors are "the single most important influencer of fashion," such that they generate a fundamental property of the fashion marketplace, namely, the seemingly random nature of the determination of a season's "it" products. Fashion editors rely on that randomness to appeal to fashion-conscious consumers

who are interested in wearing "it" products, whether for their intrinsic pleasure or for the benefits associated with signaling their fashion sense. Thus, the incentives for fashion editors to randomize their selections is clear: It generates more interest in their publications and increases their profits, because fashion-conscious consumers must access the publications to learn about the season's "it" products. We can link our analysis to the study of fashion cycles (Karni 1990; Pesendorfer 1995; Yoganarasimhan 2012), in the sense that fashion publications randomly select "it" products, and university ranking publications add randomness to their methodologies.[14] However, this literature stream has not addressed the goal of our research, namely, to demonstrate that the interaction of the ranking publication's profit motive and the prestige it creates for highly ranked universities creates an incentive to generate uncertainty in rankings and change the methodology over time.

Just as publications that use attribute-and-aggregate ranking methodologies select attribute weights, various news services choose which story attributes to cover and the weights to attach to them (at least implicitly). In that sense, our research also links to media coverage choice literature, specifically in relation to media bias (e.g., Gal-Or, Geylani, and Yildirim 2012; Mullainathan and Shleifer 2005; Xiang and Sarvary 2007). In a competitive news environment, the desire to appeal to the views of a particular group of news consumers and differentiate their products prompts news agencies to slant or add bias to their stories. Media bias also might arise in monopolistic environments, if the news service seeks to avoid offending advertisers (Reuter 2009) or if the media owners have political ambitions (Anderson and McLaren 2012). Ellman and Germano (2009) specifically identify media bias due to advertisers' interests, and Zhu and Dukes (2015) show that media bias can be even more severe in a monopoly than a competitive

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[14] The value of status goods like fashion items (universities) depends on not only product attributes but also the types of people who consume the item (attend the university) (Kuksov and Xie 2012; Veblen [1894] 1994).

environment. According to Gentzkow and Shapiro (2006, p. 282), media bias decreases in contexts in which "predictions are concrete and outcomes are immediately observable," such as sports outcomes, unlike uncertain contexts such as foreign wars or tax policies. Empirical research on media bias is beginning to emerge; in measuring the political bias of newspapers in China and assessing its causal effect on it, Qin, Strömberg, and Wu (2018) show that reforms that reduce competition (by forcing newspaper exits) influence media bias, by increasing product specialization.

Consumers in media bias studies and students in our model have different objectives though. "Biased" media consumers seek news that is consistent with their prior beliefs rather than the truth, and news services respond by slanting their coverage to match those preferences. In our model, students seek the truth when they view a publication's university rankings. Furthermore, the publisher's ranking creates prestige, an effect that is not present in media bias literature. Therefore, our findings add to that domain by showing that bias also can be a result of appealing to prestige effects in a context of conspicuous consumption (e.g., university choice).

Generalizations, Extensions, Limitations, and Further Research

We model the interaction between a publication and student stakeholders, though rankings are important to many university stakeholders. Alumni of top-ranked universities who view the rankings can brag about the standings of their alma maters; donors can use rankings as salient information about the quality of universities to predict likely returns on their investments; administrators can use rankings for marketing purposes; and faculty can use them for career decisions. In each case, the ranking creates prestige or provides information about university attributes. Therefore, our analysis and its distinction of the roles of information and prestige could readily incorporate university stakeholders other than students.

Furthermore, we concentrate on the education marketplace, but rankings publications are popular in many markets, such as for places to live (e.g., *Money*), restaurants (e.g., numerous city magazines), hotels (e.g., *Travel + Leisure*), and cooking products (e.g., *Cook's Illustrated*). The target audiences for these rankings similarly are concerned with information about product attributes (for consumers) and the prestige created (for consumers and businesses). Therefore, in other marketplaces, we anticipate that publications similarly manipulate their rankings, though possibly to a lesser degree than we find in the education market.

Our theoretical analysis underscores the need to establish empirically whether product and university rankings are only informative or provide prestige as well. Empirical analyses cite the importance of product ranks, after controlling for other factors such as product attributes, on consumer decision making (Simonsohn 2011). Luca and Smith (2009) demonstrate specifically that actual ranks in *USNWR*, after controlling for college and university attributes, influence application decisions. They suggest this direct effect of ranks is due to the salience of the information provided by the ranks. Alternatively, it could be due to the prestige they create. In a hospital setting, Pope (2009) finds that improvements in the ranks published in *U.S. News: Best Hospitals* attracts more patients. Finally, Sorensen (2007) finds that positions in *The New York Times Book Review* affect book sales. However, none of these studies examine whether the influence of the rankings on consumer choice is due to the information provided—in a Bayesian sense, related to attribute scores, or according to the salience of a simple numerical ranking—or instead is due to the prestige created by the ranking.[15]

It also would be useful to study how the prestige effect works in the presence of competition. According to literature on media bias (e.g., Mullainathan and Shleifer 2005; Zhu

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[15] In consumer product markets, Ackerberg (2001) empirically distinguishes informative and persuasive effects of advertising.

and Dukes 2015), each competing publication should seek a niche, possibly representing different ranking segments (e.g., university rankings, business school rankings, law school rankings). Alternatively, ranking publications could compete directly, as *USNWR*, *BusinessWeek*, and *Poets and Quants* do for business school rankings. In such cases, ranking publications would need to differentiate themselves to generate demand and yet maintain legitimacy to ensure consumer trust. Not all students will view every ranking (and some may not view any), so a prestige effect should exist for legitimate rankings. This effect should move the ranking away from the student optimal version (to introduce randomness), but too much randomness could threaten legitimacy perceptions, because the competing publications provide viable alternatives. Perhaps then we might anticipate the emergence of a metaranking by a publication that aggregates existing rankings to create its own version (Nagpal and Grewal 2018).

In our model, the publication maintains perpetual credibility despite adding randomness to its ranking methodology and changing it in each period. Moreover, by adding this uncertainty and changing its methodology over time, the ranking gains popularity. It does so because students want what the publication creates – prestige – and because they have no viable option to the one ranking. In reality and outside of the context of our model, a publication might be able to change its ranking methodology from year to year because it does so under the veil improving. Because university stakeholders cannot divine whether a change in methodology is for improvement, or to increase popularity, a publication can misrepresent a change in methodology to generate interest as a change to improve methodology.

For this study, we have assumed that all students have the same relative preference for product attributes (i.e., all students have the same γ^t values). If students differ in their relative preferences for product attributes, in the presence of competition among publication rankings,

different publications arguably could adjust their methodologies to cater to the needs of these different customer segments. Whether such a result would benefit or harm students depends on Segal's law (i.e., a person with one watch knows what time it is, but a person with two watches cannot be sure; Bloch 2003, p. 36). Would competition encourage rankings that are student-optimal, at least within the targeted segment? However, even if publications compete by adjusting their rankings to appeal to different customer segments, they would not be competing head-to-head, so they still may manipulate their methodologies as we have described.

We have tried to demonstrate that the apparently arbitrary changes that ranking publications make in their methodologies are not whimsical but rather are driven by a profit-oriented business logic. The problem, as we see it, will not go away on its own. Moreover, two fundamental challenges remain:

(1) When can rankings of any sort be trusted to reflect the preferences of the targeted consumers?

(2) What can be done to make rankings more trustworthy?

We hope that our work will stimulate research into both of these diagnostic and prescriptive questions and lead to innovations in the world of ranking publications.

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Appendix

Proof of Lemma 1.

Claim: For each $(\alpha^{t+1}, \beta^{t+1}) > 0$, $V_{View}^{t+1} - V_{Not}^{t+1}$ is strictly increasing in s^t. Therefore, s^{t+1} is strictly increasing in s^t.

Proof: We begin by proving the first part. The period $t + 1$ prestige effect, $\beta^{t+1} g(s^t) \rho(r_i^{t+1})$, is increasing in s^t. Furthermore, the increase in V_{View}^{t+1} is greater than the increase in V_{Not}^{t+1}. If a student does not view the ranking, let us assume she chooses university i. If she views the ranking, she chooses either university i or one that is better ranked. In the cases in which she chooses i after viewing the ranking, an increase in s^t has the same effect on her ex post utilities of viewing and not viewing the ranking. However, if viewing the ranking leads to her choice of a better-ranked university, an increase in s^t has a greater effect on her ex post utility of viewing the ranking compared with not viewing the ranking. Regarding the second part of the claim, for a given $(\alpha^{t+1}, \beta^{t+1})$, an increase in $V_{View}^{t+1} - V_{Not}^{t+1}$ could shift the student from not viewing to viewing the ranking.

With this claim, we know that $s^t(w^t) + s^{t+1}(w^{t+1}; s^t(w^t))$ is a strictly increasing transformation of $s^t(w^t)$. Similarly, s^{t+2} is strictly increasing in s^{t+1}. Therefore, s^{t+2} is strictly increasing in s^t. Continuing in this manner, $\sum_{t' \geq t} s^{t'}(\cdot)$ is a strictly increasing transformation of $s^t(w^t)$. Therefore, w^{t^*} maximizes s^t, and w^{t^*} is the period t element of the solution to

$$\max_{w^t, t=1,2,\ldots} \sum_t (w^t; s^{t-1}).$$

Proof of Lemma 2

(i) If either $H_\alpha^t(0) < 1$ and $H_\beta^t(0) = 1$ or $s^{t-1} = 0$, then

(A1) $V_{View}^t(w^t; s^{t-1}) = \alpha^t \sum_{r^t} q^t(r^t; w^t) \max_i \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t | r^t; w^t) \sum_j \gamma_j^t a_{ij}^t \, da_{11}^t \cdots da_{nm}^t$

and

(A2) $V_{Not}^t(w^t; s^{t-1}) = \alpha^t \max_i \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j \gamma_j^t a_{ij}^t \, da_{11}^t \cdots da_{nm}^t.$

Using Equations A1 and A2, a student views the ranking if and only if

(A3) $\alpha^t \geq \dfrac{c}{\left[\begin{array}{c} \sum_{r^t} q^t(r^t; w^t) \max_i \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t | r^t; w^t) \sum_j \gamma_j^t a_{ij}^t da_{11}^t \cdots da_{nm}^t \\ - \max_i \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j \gamma_j^t a_{ij}^t da_{11}^t \cdots da_{nm}^t \end{array}\right]}.$

Choosing w^t to maximize s^t is equivalent to choosing w^t to minimize the r.h.s. of Equation A3.

Because V_{Not}^t is independent of w^t, minimizing the r.h.s. of Equation A3 amounts to choosing

w^t to maximize Equation A1.

(ii) If $H_\alpha^t(0) = 1$ and $H_\beta^t(0) < 1$, and a student views the ranking, then she chooses the

top-ranked university, regardless of its identity. Her utility of viewing the ranking is

(A4) $V_{View}^t(w^t; s^{t-1}) = \beta^t g(s^{t-1})\rho(1)$

and the expected utility of not viewing the ranking is:

(A5) $V_{Not}^t(w^t; s^{t-1}) =$

$\beta^t \max_i \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t | a^t; w^t) g(s^{t-1})\rho(r_i^t) \, da_{11}^t \cdots da_{nm}^t.$

From Equations A4 and A5, we can determine that a student views the ranking if and only if:

(A6) $\beta^t \geq \dfrac{c}{g(s^{t-1})\rho(1) - \max_i \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t | a^t; w^t) g(s^{t-1})\rho(r_i^t) da_{11}^t \cdots da_{nm}^t}.$

Choosing w^t to maximize s^t is equivalent to choosing w^t to minimize the r.h.s. of Equation A6.

Because V_{View}^t is independent of w^t, minimizing the r.h.s. of Equation A6 amounts to choosing

w^t to minimize Equation A5.

Proof of Theorem 1

(i) If either $H_\alpha^t(0) < 1$ and $H_\beta^t(0) = 1$ or $s^{t-1} = 0$, then by Lemma 2, the publication's

objective is to maximize Equation 5. If the publication ranks schools according to students'

ordinal utility ranking (i.e., uses the viewing-student optimal ranking methodology), for each r^t, the student matriculates at the preferred school. Therefore, the publication's ranking maximizes Equation 5.

(ii) If $H_\alpha^t(0) = 1$ and $H_\beta^t(0) < 1$, then by Lemma 2, the publication's objective is to minimize Equation 7. If for some i' and i'',

$$\int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t|a^t; w^t) \rho(r_{i''}^t) \, da_{11}^t \cdots da_{nm}^t$$

$$\geq \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t|a^t; w^t) \rho(r_{i'}^t) \, da_{11}^t \cdots da_{nm}^t$$

then the publication can change $q^t(r^t|a^t; w^t)_{a^t \in \mathcal{A}}$ (where $\mathcal{A}$ represents the set of possible attribute profiles) to reduce Equation 7. For each

$$i''' \in \underset{i}{\operatorname{argmax}} \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t|a^t; w^t) \rho(r_{i'''}^t) \, da_{11}^t \cdots da_{nm}^t \, da_{11}^t \cdots da_{nm}^t,$$

it does so by decreasing the probability that i''' is ranked first and increasing the probability that i' is ranked first. Therefore, if the publication maximizes $V_{View} - V_{Not}$, then for each i and i',

$$\int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t|a^t; w^t) \rho(r_i^t) \, da_{11}^t \cdots da_{nm}^t$$

$$= \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j q^t(r^t|a^t; w^t) \rho(r_{i'}^t) \, da_{11}^t \cdots da_{nm}^t.$$

This equality holds if $q_R(r) = \frac{1}{n!}$. Because the student is admitted by all universities, it holds if and only if the probability that university $i, i' \in N$, is ranked first equals $1/n$. The publication uses the uniform/random ranking methodology.

Proof of Lemma 3

By induction. Assume any two periods i' and i'' are ex ante identical in terms of student preferences and university attribute scores: $\gamma^{t'} = \gamma^{t''}$, $H_\alpha^{t'} = H_\alpha^{t''}$, $H_\beta^{t'} = H_\beta^{t''}$, and $p^t = p^{t'}$. We

have $s^1 > s^0 = 0$. For t and $t-1$, assume $s^{t-1} > s^{t-2}$. Assuming the publication in period t sets $w^t = w^{t-1}$, and because the prestige effect is greater in period t than in period $t-1$, we know that $V_{View}^t - V_{Not}^t > V_{View}^{t-1} - V_{Not}^{t-1}$. Therefore, if $w^t = w^{t-1}$, then $s^t > s^{t-1}$. Because the publication has the option to set any w^t and not necessarily $w^t = w^{t-1}$, for any optimal w^t, $s^t > s^{t-1}$.

Justification of the Technical Conditions for Theorem 2

We make two technical assumptions about the distributions of α^t and β^t. First, the distributions of α^t and β^t are uniform. The effect of a greater number of students who have viewed the ranking in period $t-1$ on the publication's period t decision problem can be partitioned into two parts: the prestige effect becomes relatively more important, and the cutoff in the (α^t, β^t) space shifts, in terms of students who view the ranking and those who do not. As a result of this shift, the distributions of α^t and β^t could differ between the new and old cutoffs. If the distributions differ, the change in the ranking methodology causes varying numbers of students to switch between viewing and not viewing the ranking. This distributional effect then causes a change in the marginal value to the publication of changing its ranking methodology, inducing the publication to change its methodology. For uniform distributions of α^t and β^t, the distributions around the cutoff are independent of the position of the cutoff. Therefore, in Theorem 2, we consider uniform distributions of α^t and β^t. Note that the response by the publication to a change in the importance of prestige is analogous to a substitution effect, and the response to a change in the distributions of α^t and β^t around the cutoff is analogous to an income effect. Considering uniform distributions on α^t and β^t therefore permits us to isolate a changing prestige/substitution effect.

Second, and also related to the distributions of α^t and β^t around the cutoff between students who view the ranking and those who do not, all students with the highest possible value of the attribute scores (i.e., $\alpha^t = \bar{\alpha}$) view the ranking regardless of their values of β^t, and all students with the lowest possible attribute scores (i.e., $\alpha^t = 0$) do not, also regardless of their values of β^t. For students with the lowest and highest values of α^t, the value of α^t thus dominates β^t in driving their decision to view the ranking. This case amounts to $\hat{\alpha}(\beta^t; w^{t^*}, s^{t-1^*}) \in [0, \bar{\alpha}]$ for each $\beta^t \in [0, \bar{\beta}]$ and values of w^t and s^{t-1} that are in the neighborhood of the equilibrium values, w^{t^*} and s^{t-1^*}. Our goal with Theorem 2 is to consider a case that is not subject to distributional effects, so that we can isolate the importance of the prestige effect in period t associated with the increase in the number of students who view the ranking in period $t-1$.

Proof of Theorem 2

Consider a case in which $\gamma^t > w_1^{uniform}$, where $w_1^{uniform}$ denotes the weight attached to attribute 1 in a uniform/random ranking methodology. We can derive the function $\hat{\alpha}(\beta^t; w^{t^*}, s^{t-1^*})$ from $V_{View}^t - V_{Not}^t - c = 0$. We write X^t as the expected utility from viewing the ranking, less the expected utility from not viewing, considering only the utility of the attribute scores; we write Y^t as the expected utility from viewing the ranking less the expected utility from not viewing, considering only the prestige effect. Then,

(A7) $\hat{\alpha}(\beta^t; w^{t^*}, s^{t-1^*}) = \frac{c - \beta^t s^{t-1} Y^t(w_1^t)}{X^t(w_1^t)}.$

Using this function and assuming $h_\alpha^t = \frac{1}{\bar{\alpha}}$ and $h_\beta^t = \frac{1}{\bar{\beta}}$, we can identify the proportion of students who view the ranking in period t as

(A8) $s^t(w^t; s^{t-1}) = \int_0^{\bar{\beta}} \int_{\hat{\alpha}(\beta^t; w^{t^*}, s^{t-1^*})}^{\bar{\alpha}} \frac{1}{\bar{\alpha}} \frac{1}{\bar{\beta}} d\alpha^t d\beta^t.$

If we differentiate Equation A8 with respect to w_1^t using Leibniz' rule, we obtain the first-order condition for the publication's period t maximization problem:

(A9) $\quad \dfrac{\partial s^t\left(w^t;s^{t-1^*}\right)}{\partial w_1^t} = \dfrac{1}{\bar{\alpha}}\dfrac{1}{\bar{\beta}}\displaystyle\int_0^{\bar{\beta}} \dfrac{\partial\hat{\alpha}\left(\beta^t;w^{t^*},s^{t-1^*}\right)}{\partial w_1^t} d\beta^t = 0.$

Next, we take the total differential of Equation A9 with respect to w_1^t and s^{t-1} and set it equal to 0 (so that the first-order condition continues to be satisfied):

(A10) $\quad \displaystyle\int_0^{\bar{\beta}} \dfrac{\partial^2\hat{\alpha}\left(\beta^t;w^{t^*},s^{t-1^*}\right)}{\left(\partial w_1^t\right)^2} d\beta^t dw_1^t + \int_0^{\bar{\beta}} \dfrac{\partial^2\hat{\alpha}\left(\beta^t;w^{t^*},s^{t-1^*}\right)}{\partial w_1^t \partial s^{t-1}} d\beta^t ds^{t-1} = 0.$

Rearranging Equation A10 and noting that the second-order condition requires

$\displaystyle\int_0^{\bar{\beta}} \dfrac{\partial^2\hat{\alpha}\left(\beta^t;w^{t^*},s^{t-1^*}\right)}{\left(\partial w_1^t\right)^2} d\beta^t < 0$, we have:

(A11) $\quad sign\left|\dfrac{dw_1^t}{ds^{t-1}}\right| = sign\left|\displaystyle\int_0^{\bar{\beta}} \dfrac{\partial^2\hat{\alpha}\left(\beta^t;w^{t^*},s^{t-1^*}\right)}{\partial w_1^t \partial s^{t-1}} d\beta^t\right|.$

In evaluating the sign of Equation A11, we consider:

(A12) $\quad \dfrac{\partial^2\hat{\alpha}\left(\beta^t;w^{t^*},s^{t-1^*}\right)}{\partial w_1^t \partial s^{t-1}} = \beta^t\left(-\dfrac{\frac{\partial Y^t}{\partial w_1^t}}{X^t} + \dfrac{\beta^t Y^t \frac{\partial X^t}{\partial w_1^t}}{(X^t)^2}\right) > 0,$

because $\dfrac{\partial Y^t}{\partial w_1^t} < 0$ and $\dfrac{\partial X^t}{\partial w_1^t} > 0$ for $w_1^t \in [w_1^{uniform}, \gamma_1^t)$. Finally, combining Equations A10, A11, and A12,

(A13) $\quad \dfrac{dw_1^t}{ds^{t-1}} < 0.$

For the case in which $\gamma_1 > w_1^{uniform}$, from Lemma 3 (i.e., $s^t > s^{t-1}$) and Equation A13, we can determine that for any t and $t-1$, $w_1^{t^*} < w_1^{t-1^*}$. The proofs for the case in which $\gamma_1 < w_1^{uniform}$ and to establish that for any t and $t-1$, $w_1^{t^*} > w_1^{t-1^*}$ follow the same path as the proof for the case in which $\gamma_1 > w_1^{uniform}$.

Proof of Lemma 4

We assume that for each student, the inequality expressed in Equation 10 holds:

$$\int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j \gamma_j^t \, a_{ij}^t \, da_{11}^t \cdots da_{nm}^t >$$

$$\max_{\{i', i' \in N \backslash i\}} \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \sum_j \gamma_j^t \, a_{i'j}^t \, da_{11}^t \cdots da_{nm}^t.$$

Therefore, if $\beta^t = 0$, a student who does not view the ranking chooses to attend university 1.

When we add the prestige effect to the student's utility function ($\beta^t > 0$), if that student attends

university 1 and another university is ranked first, she would be strictly better off if she attends

university 1 and it were ranked first. If the student attends another university because it is ranked

first, she would be strictly better off if she were to switch to university 1 and it were ranked first.

Therefore, for each student who experiences a prestige effect and does not view the ranking,

expected utility is maximized if university 1 is ranked first and she attends it.

Proof of Theorem 3

The proof follows directly from Theorem 1 and Lemma 4.

Proof of Corollary 1

In the first part of Corollary 1, students are unconcerned with the prestige of university

ranks ($H_\alpha^t(0) < 1$ and $H_\beta^t(0) = 1$) or the period $t-1$ ranking has no views ($s^{t-1} = 0$). The

publication's profit-maximizing ranking methodology is $w_1^{t^*} = \gamma_1^t$, and it is socially optimal,

$w_1^{t^o} = \gamma_1^t$, because it maximizes s^t and the utility of the students who view the ranking. Without

a prestige effect, the utilities of the students who do not view the ranking are constant in the

ranking methodology, w_1^t.

In the second part of Corollary 1, some or all students are concerned with prestige

($H_\beta^t(0) < 1$) and the period $t-1$ ranking has views ($s^{t-1} > 0$). The sum of the students' utilities

from viewing and from not viewing the ranking is not maximized at $w_1^{t^*}$. A movement from $w_1^{t^*}$

in the following directions would result in an increase in the sum of the students' utilities. That is, assume $q^t((1,2), w_1^t)$ is strictly increasing in w_1^t. Therefore, university 1's expected score

$$\int_{a_{11}^t} \cdots \int_{a_{22}^t} p^t(a^t) \ (\gamma_1^t \ a_{11}^t + a_{12}^t) da_{11}^t \cdots da_{22}^t$$

is strictly increasing in w_1^t, and university 2's expected score

$$\int_{a_{11}^t} \cdots \int_{a_{22}^t} p^t(a^t) \ (\gamma_1^t \ a_{21}^t + a_{22}^t) da_{11}^t \cdots da_{22}^t$$

is strictly decreasing in w_1^t. This property, combined with the equality $q^t((1,2), w_1^t) + q^t((2,1), w_1^t) = 1$, implies that there exists a unique weight in the attribute-and-aggregate ranking methodology, $w_1^{uniform}$, for which

$$\int_{a_{11}^t} \cdots \int_{a_{22}^t} p^t(a^t) \ (\gamma_1^t \ a_{11}^t + a_{12}^t) da_{11}^t \cdots da_{22}^t$$

$$= \int_{a_{11}^t} \cdots \int_{a_{22}^t} p^t(a^t) \ (\gamma_1^t \ a_{21}^t + a_{22}^t) da_{11}^t \cdots da_{22}^t.$$

We consider two cases, based only on attribute scores: (1) students who do not view the ranking prefer university 1, and (2) students who do not view the ranking prefer university 2.

Case 1. If the students who do not view the ranking prefer university 1 to be ranked first, then from Lemma 4,

(A14) $\int_{a_{11}^t} \cdots \int_{a_{22}^t} p^t(a^t) \ (\gamma_1^t \ a_{11}^t + a_{12}^t) da_{11}^t \cdots da_{22}^t$

$$> \int_{a_{11}^t} \cdots \int_{a_{22}^t} p^t(a^t) \ (\gamma_1^t \ a_{21}^t + a_{22}^t) da_{11}^t \cdots da_{22}^t.$$

Therefore, $\gamma_1^t > w_1^{uniform}$.

We thus establish that $w_1^{t^*} \in (w_1^{uniform}, \gamma_1^t)$. Each student's net expected utility of viewing the ranking, $V_{View}^t(w^t, s^{t-1}) - V_{Not}^t(w^t, s^{t-1})$, has two linearly independent

components: the net expected utility of attribute scores and the net expected utility of prestige. Theorem 1 indicates that the first component is maximized at $w_1^t = \gamma_1^t$ and the second component is maximized at $w_1^t = w_1^{uniform}$. The first component also is strictly increasing in $w_1^t, w_1^t \in [w_1^{uniform}, \gamma_1^t]$, and the second component is strictly decreasing in $w_1^t, w_1^t \in [w_1^{uniform}, \gamma_1^t]$. Therefore, $w_1^{t^*} \in (w_1^{uniform}, \gamma_1^t)$.

Next, in Case 1, for any $\alpha^t, \beta^t > 0$, $V_{View}^t(w^t; s^{t-1}) + V_{Not}(w^t; s^{t-1})$ is maximized at $w_1^t, w_1^t \in (\gamma_1^t, \infty)$. We demonstrate in Theorem 1 that a student's gross expected utility of viewing the ranking $V_{View}^t(w^t; s^{t-1})$ is maximized at $w_1^{t^*} = \gamma_1^t$ and strictly decreasing in $w_1^t, w_1^t \in (\gamma_1^t, \infty)$. From Equation A14, we determine that $V_{Not}^t(w^t; s^{t-1})$ is strictly increasing in $w_1^t, w_1^t \in (\gamma_1^t, \infty)$. Therefore, $\alpha^t, \beta^t > 0$, $V_{View}^t(w^t; s^{t-1}) + V_{Not}(w^t; s^{t-1})$ is maximized at $w_1^t, w_1^t \in (\gamma_1^t, \infty)$. Because s^t is maximized at $w_1^{t^*}, w_1^{t^*} \in (w_1^{uniform}, \gamma_1^t)$ and for any $\alpha^t, \beta^t > 0$, $V_{View}^t(w^t; s^{t-1}) + V_{Not}(w^t; s^{t-1})$ is maximized at $w_1^t, w_1^t \in (\gamma_1^t, \infty)$, we also know that

(A15) $w_1^{t^*} < w_1^{t^{aso}}$.

Equation A15, combined with our condition that $q^t((1,2), w_1^t)$ is strictly increasing in w_1^t, thus implies that $q^t\left((1,2), w_1^{t^{aso}}\right) > q^t\left((1,2), w_1^{t^*}\right) > \frac{1}{2}$.

Case 2. The proof of Case 2 is analogous to the proof of Case 1.